Sampling and Prefiltering Effects on Blind Equalizer Design

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***Abstract*—In the development of equalization algorithms for unknown channels, the effects of the sampling rate and the analog receive prefilter prior to discretization of the received signal are often overlooked. In this paper, these effects are investigated. The relationship between the fractionally spaced output samples of a noise-limiting prefilter and the symbol spaced output samples of a matched filter is studied for both the time-invariant and the time-varying channels. It is shown that the prefilter and the sampling rate can have significant effects on blind equalization algorithms. Thus, this paper provides a common framework for comparing different blind algorithms that are studied in the literature with different sampling rates. A case study of the well-known subspace method for blind channel identification is presented. The effects of the noise color due to the prefilter on equalizers is investigated, and the sensitivity of the truncation of the overall channel impulse response in terms of the mean squared error (MSE) performance criterion is investigated through numerical examples.**

***Index Terms*—Equalizers, intersymbol interference, matched filters, maximum likelihood detection, noise, time-varying channels, white noise.**

I. INTRODUCTION

A MAJORITY of equalization algorithms proposed in the literature for unknown channels start with a discrete time quadrature amplitude modulation (QAM) baseband system model. Many of these algorithms are blind algorithms that can be broadly grouped into two classes depending on the sampling rate of the receiver: symbol spaced blind algorithms and fractionally spaced blind algorithms. It is commonly assumed that after symbol spaced or fractionally spaced sampling, the discrete noise sequence remains discretely white.

For symbol spaced algorithms, the functional equivalence of the discrete-time channel model to the actual continuous-time channel has been largely overlooked. This possible misconception may be traced to an incorrect interpretation of the work of Forney [1]. There, it is shown that for an *a priori* known channel, discretization by using an analog filter matched to the overall channel impulse response and sampling its output at the symbol rate provides a set of sufficient statistics for maximum likelihood sequence estimation of the transmitted sequence. This result has been used as the basis for essentially all works in equalization in justifying (at least implicitly) the use of symbol rate discrete time models. However, in most wireless communication systems and nonfixed wireline communication systems, the channel is not known *a priori*, and the matched filter is, therefore, unknown. As a result, a set of sufficient statistics cannot be guaranteed by discrete symbol rate samples at the receiver filter output [2]. In short, it is incorrect to assert that any symbol rate discrete time channel model is functionally equivalent to an actual physical model.

The developments in fractionally spaced equalization algorithms have presented an alternative to a front-end analog matched filter. Although fractionally spaced equalizers have been studied for a long time [3]–[6], blind fractionally spaced equalization algorithms and their analysis are relatively more recent [7]–[10]. These algorithms operate on samples obtained at a rate higher than the symbol rate, usually satisfying the Nyquist sampling criterion. Although there is an increase in the complexity of the analog-to-digital conversion process, these equalizers offer potentially significant advantages over the conventional symbol spaced equalizers in terms of lower timing phase sensitivity [4], [5], reduced noise enhancement, and additional statistical information for channel identification [11], [12]. In fact, the use of fractional sampling often allows the blind identification of unknown channels based only on second-order statistics by providing a discrete single input multiple output (SIMO) channel model.

Vachula and Hill [13] showed that fractionally spaced sampling in the receiver front end alleviates the need to use an analog prefilter matched to the overall channel impulse response to obtain a set of sufficient statistics. However, their receiver has limited practical appeal because it depends on the use of an ideal lowpass filter. It was shown by Chugg and Polydoros [2] that fractionally spaced sampling at the output of a filter matched[1] to the known pulse shaping filter in the transmitter provides a set of sufficient statistics. Hence, fractionally spaced sampling can lead to a meaningful discrete time model without relying on *a priori* knowledge of the channel. While the work in [2] rigorously addresses a very important issue, we show that it is restrictive and incompatible with the discrete time models assumed in many blind equalization algorithms such as those

[1]This work also implicitly assumes a coherent sampling at the receiver, but the generalization to arbitrary sampling phase (as we do here) is straightforward.

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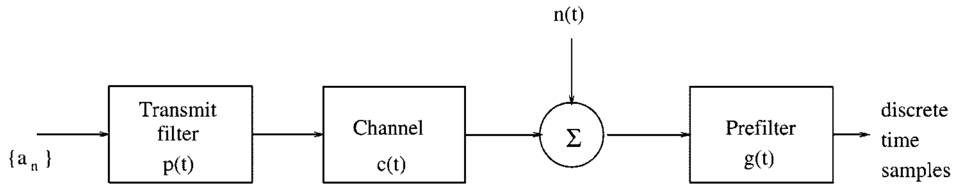

Fig. 1. Block diagram showing part of a data communication system.

found in [7], [9]. The prefilter presented in [14] does not address the noise coloring problem in the context of fractionally spaced blind equalization algorithms.

In this paper, we highlight the critical role that sampling and prefiltering play in the justification and development of blind algorithms. In the literature, many comparisons of the fractionally spaced blind equalizers have been done unfairly using different sampling rates [15], [16]. They tend to create the impression that faster sampling will generate better identification results. This work is aimed at studying the validity of modeling under different front-end analog filterings, sampling rates, and noise models by bringing the analysis of blind algorithms under a common framework.

The contributions of this paper are as follows.

1) We generalize the results in [2] and [13] to show that an *arbitrary analog prefilter* with spectral support at least equal to the signal part of the received signal, in conjunction with fractionally spaced sampling, yields sufficient statistics for maximum likelihood sequence estimation and related criteria. The relationship between the symbol spaced output samples of a matched filter and the fractionally spaced output samples of a noise-limiting prefilter is studied for both time-invariant and time-varying channels.
2) We demonstrate that an analog prefilter in conjunction with fractionally spaced sampling is *robust to sampling phase offsets* that remove the requirement to synchronize with the symbol timing implicit in [2], [13], and [17].
3) We show the relationship of the analog prefilter, the sampling rate, and the noise model assumed in the equalization algorithms and thus establish the analog prefilter needed to satisfy the white noise SIMO blind identification model. A case study of the well-known subspace method is presented.
4) We demonstrate the effect of the noise color on linear and decision feedback equalizers. The sensitivity of the truncation of the overall channel impulse response length in colored noise is shown through numerical examples.

II. Symbol Spaced Equalization

Consider a transmitted waveform

$$s(t) = \sum_n a_n p(t - nT)$$

where

- $\{a_n\}$ input data sequence;
- T symbol interval;
- $p(t)$ impulse response of the pulse shaping filter;
- n symbol spaced time index.

As shown in Fig. 1, the signal $s(t)$ passes through a channel with impulse response $c(t)$ resulting in an output signal

$$r(t) = \sum_n a_n u(t - nT) + n(t) \tag{1}$$

where $u(t) = c(t) * p(t) = \int_{-\infty}^{\infty} c(\tau)p(t-\tau)\,d\tau$ is the finite energy convolution of the response of the channel with the input signal pulse $p(t)$, and $n(t)$ is additive white noise.

Let the front-end analog filter of the receiver be given by $g(t)$. It is well known that a matched filter having an impulse response $g(t) = u^*(-t)$, where "$*$" denotes complex conjugation, is the front end of a receiver designed for optimum detection of the input data sequence [1], [18]. Symbol rate samples $\{y_n\}$ of the output of the matched filter are given by[2]

$$\begin{aligned} y_n &= r(t) * u^*(-t)|_{t=nT} \\ &= \int_{\tau\in\Omega} r(\tau)u^*(\tau - t)\,d\tau \bigg|_{t=nT} \end{aligned} \tag{2}$$

where Ω is the observation interval. These symbol rate samples $\{y_n\}$ form a set of sufficient statistics[3] for maximum likelihood sequence estimation (MLSE) and provide discrete time stationary samples as input to the equalizer. Clearly, the key condition needed for deriving sufficient statistics from symbol rate samples is that the overall response $u(t)$ is known *a priori*. When the channel is not known *a priori*, as in the case of blind equalizers or whenever a training sequence is employed for channel identification, a matched filter cannot be used, and we cannot rely on (2) in our receiver design to provide a set of sufficient statistics [2]. As a result, the performance of symbol-spaced equalizers becomes dependent on the choice of the prefilter.

III. Fractionally Spaced Equalization

A. Background

Because symbol rate samples of the prefilter output may not preserve sufficient statistics for channel equalization (since the channel is usually unknown and the correct sampling phase is unknown), we now investigate whether faster sampling rate will admit sufficient statistics and whether this is feasible for implementation. The objective will be to show how the ideal symbol spaced samples in (2) that contain sufficient statistics can be expressed in terms of fractionally spaced samples. This will imply

[2]Note that these samples are taken with correct sampling phase, and any timing offset Δ (which is not a multiple of the symbol period T) in the form of $y(nT + \Delta)$ usually implies loss of optimality.

[3]In fact, these symbol rate samples obtained at the output of a matched filter provide sufficient statistics for a much broader class of optimality criteria that subsumes the MLSE case [18].

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that the fractionally spaced samples also contain sufficient statistics, but they do so under far more lax requirements than those implicit in (2).

We now consider the case when the input signal pulse shaping filter $p(t)$ is bandlimited to $|f| \leq 1/2T_r$, where $T_r = T/r$ with integer $r > 1$. In most QAM and PAM systems, $r = 2$ can be chosen when the transmitted pulse has a roll-off factor so that there is no signal energy beyond $|f| > 1/T$.[4]

Assuming that $u(t)$ is bandlimited within $|f| < 1/2T_r$, we can expand $u(t)$ in terms of its $1/T_r$ rate sampling with some arbitrary timing offset Δ using the sampling theorem, i.e.,

$$u(t) = \sum_{m=-\infty}^{\infty} u_m \operatorname{sinc}\left(\frac{1}{T_r}(t - mT_r - \Delta)\right) \tag{3}$$

with fractionally spaced coefficients

$$u_m = u(t)|_{t=mT_r+\Delta}$$

where m is the fractionally spaced time index.[5] Note that the arbitrary sampling phase offset Δ is relative to the symbol spaced optimum sampling phase implicit in (2). Therefore, utilizing (3), the symbol spaced matched filter output samples $\{y_n\}$, given by (2), can be written as

$$\begin{aligned} y_n &= \int_{\tau\in\Omega} r(\tau) \sum_{m=-\infty}^{\infty} u_m^* \\ &\quad \times \operatorname{sinc}\left(\frac{1}{T_r}(\tau - nT - mT_r - \Delta)\right) d\tau \\ &= \sum_{m=-\infty}^{\infty} u_m^* \int_{\tau\in\Omega} r(\tau) \\ &\quad \times \operatorname{sinc}\left(\frac{1}{T_r}(\tau - (nr+m)T_r - \Delta)\right) d\tau \end{aligned} \tag{4}$$

where we have substituted $T = rT_r$.

B. Sufficient Statistics Prefiltering

The question arises about the existence and classification of general prefilters $g(t)$ that yield sufficient statistics under fractionally spaced sampling. It is highly desirable that such sufficient statistics are maintained under arbitrary sampling phase Δ. Partial answers to this question in the form of explicit filters (special cases) and when the sampling phase is zero have been presented in the literature [2], [13]. Here, we generalize these results and give a simple demonstration that essentially any filter $g(t)$ that is invertible within the signal bandwidth and any sampling phase can be used.

The key idea in what follows is to factorize the fractionally spaced overall channel impulse response $\{u_m\}$ into two impulse responses—$\{d_m\}$ and $\{q_m\}$—given by the discrete convolution

$$u_m = d_m * q_m. \tag{5}$$

[4]Indeed, the so-called excess bandwidth, which is the bandwidth of the channel beyond $1/2T$, is typically only 10 to 30% of $1/2T$ and, therefore, considerably less than $1/T$ [13].

[5]We use the convention of n indexing symbol-spaced samples and m indexing fractionally spaced samples.

Given $\{q_m\}$, let the receiver prefilter $g(t)$ be matched to $q(t)$, as in

$$g(t) = q^*(-t) \tag{6}$$

where

$$q(t) = \sum_{m=-\infty}^{\infty} q_m \operatorname{sinc}\left(\frac{1}{T_r}(t - mT_r)\right)$$

is a filter bandlimited to $|f| < 1/2T_r$. Its response to the channel output $r(t)$ (1) is given by

$$v(t) = r(t) * q^*(-t)$$

whose fractionally spaced samples are

$$\begin{aligned} v_m &= v(t)|_{t=mT_r+\Delta} \\ &= \int_{\tau\in\Omega} r(\tau) q^*(\tau - t)\, d\tau|_{t=mT_r+\Delta} \\ &= \sum_{k=-\infty}^{\infty} q_k^* \int_{\tau\in\Omega} r(\tau) \\ &\quad \times \operatorname{sinc}\left(\frac{1}{T_r}(\tau - (k+m)T_r - \Delta)\right) d\tau. \end{aligned} \tag{7}$$

This expression is analogous to the symbol spaced samples of the matched filter output (4).

Using (5), we can express (4) in terms of the fractionally spaced samples (7)

$$y_n = v_m * d_{-m}^*|_{m=nr} = \sum_{m=-\infty}^{\infty} d_m^* v_{nr+m} \tag{8}$$

where we recall that $\{y_n\}$ is the set of sufficient statistics obtained from sampling the matched filter output at the symbol rate with the optimum sampling phase. Equation (8) shows that $\{y_n\}$ and, hence, sufficient statistics can be extracted from the Nyquist (fractional) rate samples $\{v_m\}$ at output of prefilter $g(t) = q^*(-t)$ and that such a relationship exists for all Δ. Note that the discrete convolution in (8) (and $r:1$ decimation) is a theoretical construction only and need not be explicitly implemented as a filter in the receiver. Such a digital filter (8), which maps the fractionally spaced samples to the outputs of the symbol spaced matched filter output samples (obtained with correct sampling phase), will be referred to as a sufficient statistics filter.

C. Frequency Domain Interpretation

In the previous subsection, the "choice" of prefilter (6) appears to be arbitrary. Here, we use the frequency domain to show that there are only very weak conditions on the choice of prefilter, and therefore, in a practical sense, it can be arbitrary.

Denote $D(z) = \sum_m d_m z^{-m}$, $Q(z) = \sum_m q_m z^{-m}$, and $U(z) = \sum_m u_m z^{-m}$. Thus, the sufficient statistics filter

$$D(z) = \frac{U(z)}{Q(z)} \tag{9}$$

will exist as long as $Q(z)$ does not have extra zeros on the unit circle that are not zeros of $U(z)$ (where the zeros need to

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be counted with multiplicity). In other words, *any* bandlimited filter impulse response $g(t)$ (6) can be used as a prefilter as long as it does not have additional spectral nulls besides those of the true channel (nor greater multiplicity of the order of any zero if it does correspond). From a practical perspective, there is no sense in having a prefilter that has spectral nulls for the reasons that i) the channel is unknown, and therefore, any zeros of the channel would be unknown; and ii) it would not be a robust filter under mismodeling.

This simple zero condition is sufficient to realize (5) and to retain the sufficient statistics for channel equalization. This contrasts with [2] and [13], wherein specific prefilters need to be implemented. The discretization analog prefilter can be essentially arbitrary, and any matching can be done completely digitally without approximation.

D. Extension to Time-Varying Channels

The sufficient statistics prefiltering results can be extended to time-varying channels. Let $c(t, \tau)$ be the impulse response of a time-varying channel, where $c(t, \tau)$ denotes the response of the channel at time t due to an impulse applied at time $t-\tau$ [19]. We show here that the sufficient statistics prefilter in this case is a time-varying filter with a spectral support $|f| \le f_s + f_D$, where the transmit filter $p(t)$ is bandlimited to $|f| \le f_s$, and $c(t, \tau)$ is bandlimited in t within $|f| \le f_D$. The received signal $r(t)$ equals

$$r(t) = \sum_n a_n u(t, t-nT) + n(t) \tag{10}$$

where $u(t, t-nT) = \int_{-\infty}^{\infty} p(t-nT-\tau)c(t, \tau)\, d\tau$ is the convolution of the time-invariant transmit pulse $p(t)$ with $c(t, \tau)$. We can express this in a more general form as

$$u(t, \xi) = \int_{-\infty}^{\infty} p(\xi-\tau)c(t, \tau)\, d\tau. \tag{11}$$

Since the transmit pulse $p(t)$ is bandlimited to $|f| \le f_s$, for a given t, $u(t, \xi)$ is also bandlimited in ξ within $|f| \le f_s$ since it is obtained as a convolution of $p(\xi)$ and $c(t, \xi)$ in ξ. Therefore, using the sampling theorem, the response $u(t, \xi)$ at a given time t can be expressed as

$$u(t, \xi) = \sum_m u_m(t) \operatorname{sinc}\left(\frac{1}{T_s}(\xi - mT_s)\right) \tag{12}$$

where $f_s = 1/2T_s$, and

$$u_m(t) = u(t, \xi)|_{\xi = mT_s}.$$

The coefficients $\{u_m(t)\}$ provide the time variations of $u(t, \xi)$ at $\xi = mT_s$. Since $p(\xi - \tau)$ in (11) is independent of t, and, therefore, if $c(t, \tau)$ is bandlimited in t within $|f| \le f_D$, then for a given ξ, $u(t, \xi)$ is also bandlimited in t within $|f| \le f_D$. Therefore, from (12), $u_m(t)$ is bandlimited to $|f| \le f_D$ for a given ξ. Using $\xi = t - nT$, we get, from (12)

$$u(t, t-nT) = \sum_m u_m(t) \operatorname{sinc}\left(\frac{1}{T_s}(t - nT - mT_s)\right). \tag{13}$$

Let $\psi_{m,n}(t - nT - m\gamma T) = u_m(t) \operatorname{sinc}((1/T_s)(t - nT - mT_s))$, where $\gamma = T_s/T$. Then, $\psi_{m,n}(t-(n+m\gamma)T)$ is bandlimited to $|f| \le f_s + f_D$ since it is a product of $u_m(t)$, which is bandlimited to $|f| \le f_D$, and the sinc function, which is bandlimited to $|f| \le f_s$. Therefore, using the sampling theorem

$$\psi_{m,n}(t-(n+m\gamma)T) = \sum_k \psi_{m,n,k} \operatorname{sinc}\left(\frac{1}{T_r}(t-(n+m\gamma)T - kT_r - \Delta)\right) \tag{14}$$

where Δ is an arbitrary sampling phase offset. The sampling rate $1/T_r$ satisfies $1/T_r \ge 2(f_s + f_D)$, and $\psi_{m,n,k} = \psi_{m,n}(t - (n+m\gamma)T)|_{t-(n+m\gamma)T = kT_r + \Delta}$. Therefore

$$u(t, t-nT) = \sum_m \sum_k \psi_{m,n,k} \times \operatorname{sinc}\left(\frac{1}{T_r}(t-(n+m\gamma)T - kT_r - \Delta)\right).$$

The matched filter output samples in this time-varying channel case are given by

$$\begin{aligned} y_n &= \int_{t\in\Omega} r(t) u^*(t, t-nT)\, dt \\ &= \sum_m \sum_k \psi^*_{m,n,k} \int_{t\in\Omega} r(t) \\ &\quad \times \operatorname{sinc}\left(\frac{1}{T_r}(t-(nr+m\gamma+k)T_r - \Delta)\right) dt. \end{aligned} \tag{15}$$

This expression is similar to (4). The sequence $\{\psi_{m,n,k}\}$ can be factored into two sequences: $\psi_{m,n,k} = d_{m,n,k} * q_k$, as in (5). Therefore, a similar prefilter $q^*(-t)$ can be used, but the bandwidth of this filter in this case is $|f| \le 1/2T_r$, which is more than the bandwidth of the filter in (6). The sampling rate must be larger than $2(f_s + f_D)$. In the rest of the paper, the channel is assumed to be time invariant.

IV. Discrete Noise Spectrum under Sampling and Prefiltering

A. Uniformly Sampled Noise Spectra

In the following, we willl assume that the received signal is bandlimited to $|f| \le 1/T$ so that samples at the rate r/T, $r \ge 2$ provide sufficient statistics. The objective is to study whether there is an advantage for equalization in selecting r larger than the minimum that is necessary to achieve sufficient statistics (in this case $r = 2$). Now obviously, as far as optimal detection is concerned, higher rates provide no additional information, so such a study would be futile. The issue here is that the equalization structures employed in blind equalization are typically suboptimal, and the algorithms that operate on those structures are also suboptimal.

In the blind equalization literature, different sampling rates are often used in simulation of blind equalization systems. However, the white noise assumption persists in almost every case, regardless of the choice of r (sampling rate). In this section, we show that the prefilter and the sampling rate affect the statistics

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of the noise samples. Consequently, the white noise assumption and the actual value of the noise variance must be carefully examined in performance comparisons of fractionally spaced blind equalizers.

Given a prefilter $g(t)$, the analog noise at the receive filter output is given by

$$w(t) = \int_{-\infty}^{\infty} n(\tau) g(t-\tau)\, d\tau. \tag{16}$$

If the filter output is sampled at instants $kT_r + \Delta$, then the sampled noise is

$$w_{k,\,r} = w(kT_r) = \int_{-\infty}^{\infty} n(\tau) g(kT_r + \Delta - \tau)\, d\tau. \tag{17}$$

The autocorrelation sequence of the noise samples is

$$\begin{aligned} R_{m-k,\,r} &= E\{w_{m,\,r} w^*_{k,\,r}\} \\ &= \frac{N_0}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g\left(mT_r - \Delta - \tau_1\right) \\ &\quad \times g^*\left(kT_r - \Delta - \tau_2\right) \delta\left(\tau_1 - \tau_2\right)\, d\tau_1\, d\tau_2 \\ &= \frac{N_0}{2} \int_{-\infty}^{\infty} g(mT_r + \Delta - \tau) g^*(kT_r + \Delta - \tau)\, d\tau \\ &= \frac{N_0}{4\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 e^{j(m-k)\omega T_r}\, d\omega. \end{aligned} \tag{18}$$

Therefore, the power spectral density of the output noise samples is given by

$$\begin{aligned} S_r(\omega) &= \sum_{m=-\infty}^{\infty} R_{m,\,r} e^{-j\omega m T_r} \\ &= \frac{N_0}{4\pi} \int_{-\infty}^{\infty} |G(\nu)|^2 \sum_{m=-\infty}^{\infty} e^{jm(\nu-\omega)T_r}\, d\nu \\ &= \frac{N_0}{4\pi} \int_{-\infty}^{\infty} |G(\nu)|^2 \cdot \frac{2\pi}{T_r} \sum_{m=-\infty}^{\infty} \delta\left(\nu - \omega - \frac{m2\pi}{T_r}\right) d\nu \\ &= \frac{N_0}{2T_r} \sum_{m=-\infty}^{\infty} \left| G\left(\omega + \frac{m2\pi}{T_r}\right)\right|^2. \end{aligned} \tag{19}$$

Hence, with $T_r = T/r$, the sampled noise spectrum is

$$S_r(\omega) = \frac{rN_0}{2T} \sum_{m=-\infty}^{\infty} \left| G\left(\omega + \frac{m2\pi r}{T}\right)\right|^2. \tag{20}$$

When symbol rate samples are taken with $r = 1$, the sampled noise $w_{k,\,1}$ has power spectrum

$$S_1(\omega) = \frac{N_0}{2T} \sum_{m=-\infty}^{\infty} \left| G\left(\omega + \frac{m2\pi}{T}\right)\right|^2. \tag{21}$$

It is clear that the oversampling factor r and the prefilter response $G(\omega)$ both contribute to the sampled noise spectrum. Hence, their effects on the performance of blind equalization algorithms should not be overlooked.

B. Fractional Rate (Uncorrelated) White Noise Filter

In many blind equalization algorithms [7]–[9], there is often the underlying assumption that the oversampled noise sequence is white or has a nonsingular covariance matrix allowing whitening. This assumption has its roots in array processing where the sequences at different antenna elements are assumed to be white. In the case of fractionally spaced samples, as the data sequence can be divided into r subsequences, each noise subsequence is white, but it is correlated with the $r-1$ other noise subsequences. Therefore, the requirement for the white noise model assumption needs investigation. Specifically, the following question is important: For a fixed receiver prefilter, does the sampling rate affect performance? Note that the sampled noise may be colored if the sampling rate is increased and whitening the noise with a wider bandwidth prefilter increases the noise power.

As we showed in Section III-B, the receiver prefilter should have no spectral nulls in the channel passband. In additon, if the equalizer design requirement is for white sampled noise $w_{k,\,r}$, a good choice of lowpass prefilter is the root raised cosine (RRC) filter with roll-off over $r/2T$. The impulse response of an RRC filter with a roll-off over $1/2T$ is known [20]. A generalization of this expression for an RRC filter with a roll-off over $(1+\beta)/2T$, where β is a constant, gives the following expression for the impulse response.

$$\begin{aligned} g(t) = &\frac{(1-\alpha+\beta)}{T} \operatorname{sinc}\left(\frac{(1-\alpha+\beta)t}{T}\right) \\ &+ \frac{\alpha}{T} \operatorname{sinc}\left(\frac{\alpha t}{T} + \frac{1}{4}\right) \cos\left(\frac{(1+\beta)\pi t}{T} + \frac{\pi}{4}\right) \\ &+ \frac{\alpha}{T} \operatorname{sinc}\left(\frac{\alpha t}{T} - \frac{1}{4}\right) \cos\left(\frac{(1+\beta)\pi t}{T} - \frac{\pi}{4}\right). \end{aligned} \tag{22}$$

The constant β controls the bandwidth of the filter. We will call this prefilter a fractional rate uncorrelated noise (pre)filter (FRUNF). For a fractionally spaced white noise model at the output of the prefilter, we need $\beta = r - 1$. However, it should be noted that in an effort to guarantee that the fractional rate noise is white, the prefilter may be letting additional noise into the equalizer. The frequency response of a transmit filter $p(t)$ and the FRUNF filter for $r = 2$ and $r = 3$ is shown in Fig. 2.

C. Symbol Rate Uncorrelated Noise Prefilter

Clearly, the bandwidth of FRUNF is much wider than the channel signal bandwidth for higher sampling rates. In effect, the use of a FRUNF as receiver prefilter admits channel noise at frequencies where there is no signal content. This indicates that in an effort to guarantee that the fractional rate noise is white, the receiver must let additional noise into the equalizer. Hence, the fractional white noise assumption may have sacrificed system performance for the sake of having a simplifying noise assumption for the development of the blind algorithm. We now show how this condition can be relaxed using a different prefilter.

From Fig. 2, it is clear that the channel bandwidth is limited within $|f| \leq 1/T$. Hence, to reduce the channel noise effect, it would make sense to design a prefilter that has bandwidth equal to the channel bandwidth without admitting additional noise. As has been shown in Section III-B, sufficient statistics by fractional sampling are preserved as long as the prefilter does not introduce additional nulls in the channel passband. It is, there-

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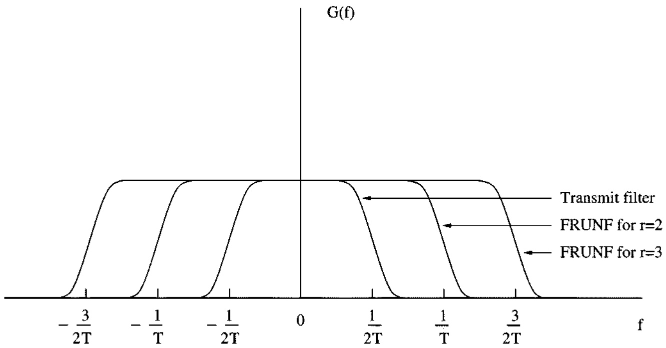

Fig. 2. Frequency response of the transmit filter and the fractional rate uncorrelated noise filter.

fore, clear that the prefilter can use the RRC filter with roll-off over $1/2T$ so that $\beta = 0$ in (22).

Nevertheless, (20) shows that the use of a lowpass RRC filter with roll-off frequency $1/2T$ will not generate a white fractional rate noise. In fact, the noise will be colored, yet with a lower variance. An obvious question we must answer is whether fractional rate blind equalization performs better under white discrete noise with higher variance or under colored discrete noise. The answer to this question will have significant impact on the design of practical blind equalizers for QAM communication systems.

V. Effects of Noise Model on Equalization Algorithms

Many blind algorithms estimate the channel impulse response and then perform equalization [21]. If the equalizer taps are computed from the estimated channel impulse response, then their performance is affected by the noise model assumed.

A. Linear Equalizers

Consider a fractionally spaced linear equalizer of length rN. The output symbol spaced samples $\{\alpha_n\}$ of the equalizer are

$$\alpha_n = \mathbf{f}^H(\mathbf{Ha} + \mathbf{w}) \tag{23}$$

where $\mathbf{f} = [f_0, f_1, \cdots, f_{rN-1}]^T$ is the fractionally spaced equalizer tap vector.[6] The matrix $\mathbf{H}$ is the convolution matrix of the overall fractionally spaced channel impulse response, consisting of the transmit filter $p(t)$, the multipath channel $c(t)$, and the receive prefilter $g(t)$. The jth row of $\mathbf{H}$ is of the form $[\mathbf{0}_{1,\xi}, \mathbf{h}^{(m)T}, \mathbf{0}_{1,\eta}]$, $\xi = \lfloor j/r \rfloor$, $m = r - 1 - j \bmod r$, $\eta = N - 1 - \xi$, $0 \leq j < rN$. The mth subchannel response vector $\mathbf{h}^{(m)}$ equals $[h_m, h_{m+r}, \cdots, h_{m+r(L-1)}]^T$, and $\mathbf{h} = [h_0, h_1, \cdots, h_{rL-1}]^T$ is the overall channel impulse response vector. The symbol vector $\mathbf{a}$ equals $[a_n, \cdots, a_{n-d}, \cdots, a_{n-(N+L-1)+1}]^T$. The noise $\mathbf{w}$ is the fractionally spaced noise sample vector at the output of the filter $g(t)$. The linear equalizer $\mathbf{f}$ minimizes the mean squared error (MSE) $E[|\alpha_n - a_{n-d}|^2]$ so that the optimal solution for the equalizer taps is

$$\mathbf{f} = \left(\mathbf{HH}^H + \mathbf{R}_w\right)^{-1}\mathbf{h}_d \tag{24}$$

where $\mathbf{h}_d$ denotes the dth column of $\mathbf{H}$, and it corresponds to the set of achievable delays $[0, N+L-2]$. The parameter d is referred to as the decision delay. The noise correlation matrix is $\mathbf{R}_w = E[\mathbf{ww}^H]$. Then, the following cases can be considered.

Case 1: The algorithm assumes white noise model, and the true noise samples are white. In that case, the MSE becomes

$$\text{MSE} = 1 - \mathbf{h}_d^H\left(\mathbf{HH}^H + \sigma_w^2\mathbf{I}\right)^{-1}\mathbf{h}_d.$$

The computation of the equalizer taps by this approach requires an estimate of the noise variance σ_w^2. Observe that if the noise power spectral density is N_0, for a sampling rate r, $\sigma_w^2 = N_0(r/T)$. Therefore, using $\mathbf{HH}^H + \sigma_w^2\mathbf{I} = \mathbf{U}(\mathbf{\Lambda} + \mathbf{\Sigma})\mathbf{U}^H$, where the columns of $\mathbf{U}$ and the diagonal elements of $\mathbf{\Lambda}$ are the eigenvectors and the eigenvalues of $\mathbf{HH}^H$, respectively, we have

$$\text{MSE} = 1 - \sum_i \frac{|\mathbf{u}_i^H\mathbf{h}_d|^2}{\lambda_i + \sigma_w^2} = 1 - \sum_i \frac{|\mathbf{u}_i^H\mathbf{h}_d|^2}{\lambda_i + r\left(\frac{N_0}{T}\right)}. \tag{25}$$

It will be observed in our numerical results (Fig. 6) that for a given delay d, an increase in the sampling rate does not reduce the MSE, and one of the reasons for this observation is the presence of r in the denominator of (25).

Case 2: The algorithm assumes colored noise model, and the true noise samples are colored. The MSE in this case is

$$\text{MSE} = 1 - \mathbf{h}_d^H(\mathbf{HH}^H + \mathbf{R}_w)^{-1}\mathbf{h}_d.$$

Note that the computation of the equalizer taps by this method requires the knowledge of the noise correlation matrix $\mathbf{R}_w$, which can be obtained from the prefiltering response.

Case 3: The algorithm assumes white noise model, but the true noise samples are colored. The MSE then becomes

$$\text{MSE} = E\left[\left|\mathbf{f}^H(\mathbf{Ha} + \mathbf{w}) - a_{n-d}\right|^2\right].$$

Using $\mathbf{f} = (\mathbf{HH}^H + \sigma_w^2\mathbf{I})^{-1}\mathbf{h}_d$, where σ_w^2 is the estimated noise variance assuming white noise model, the MSE expression can be simplified to

$$\begin{aligned}\text{MSE} = {} & 1 - \mathbf{h}_d^H\left(\mathbf{HH}^H + \sigma_w^2\mathbf{I}\right)^{-1}\mathbf{h}_d \\ & + \mathbf{h}_d^H\left(\mathbf{HH}^H + \sigma_w^2\mathbf{I}\right)^{-1}\left(\mathbf{R}_w + \sigma_w^2\mathbf{I}\right) \\ & \times \left(\mathbf{HH}^H + \sigma_w^2\mathbf{I}\right)^{-1}\mathbf{h}_d.\end{aligned}$$

The fourth case of assuming a colored noise model when the true noise samples are white is not important from a practical viewpoint.

With a view to understanding the eigenstructure, the received sample vector $\mathbf{y}$ may be expressed as

$$\mathbf{y} = \mathbf{Ha} + \mathbf{w} = \tilde{\mathbf{H}}\tilde{\mathbf{a}}$$

[6] The notations $(\cdot)^T$, $(\cdot)^H$, $E[\cdot]$, $\lfloor\cdot\rfloor$ and $a \bmod b$ denote transpose, Hermitian transpose, expectation, floor function, and modulus, respectively. The notations $\mathbf{0}_{a,b}$, $\mathbf{I}_m$ denote an $a \times b$ zero matrix and an $m \times m$ identity matrix, respectively.

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where $\tilde{\mathbf{H}} = [\mathbf{H}, \mathbf{v}_1, \cdots, \mathbf{v}_{rN}]$, and $\tilde{\mathbf{a}} = [\mathbf{a}^T, b_1, \cdots, b_{rN}]^T$. The vectors $\{\mathbf{v}_i\}$ are the eigenvectors of the correlation matrix $\mathbf{R}_w$, and $\{b_i\}$ are zero mean uncorrelated random variables [Karhunen–Loeve (KL) expansion]. Then, the MSE becomes

$$\text{MSE} = E\left[\left|\mathbf{f}^H\tilde{\mathbf{H}}\tilde{\mathbf{a}} - a_{n-d}\right|^2\right]. \tag{26}$$

Assuming the elements of $\tilde{\mathbf{a}}$ to be uncorrelated, the MSE becomes

$$\text{MSE} = \left|1 - \mathbf{f}^H\mathbf{h}_d\right|^2 + \sum_{i=0,\, i\neq d}^{K}\left|\mathbf{f}^H\mathbf{h}_i\right|^2 + \sum_{i=1}^{rN}\lambda_i\left|\mathbf{f}^H\mathbf{v}_i\right|^2 \tag{27}$$

where $K = L + N - 2$. The MSE consists of three terms.

1) The first term $|\,1 - \mathbf{f}^H\mathbf{h}_d\,|^2$ represents the power of the offset of the equalized symbol from unity.
2) The second term $\sum_{i=0,\, i\neq d}^{K}|\,\mathbf{f}^H\mathbf{h}_i\,|^2$ is the contribution to the MSE due to residual intersymbol interference (ISI).
3) The third term $\sum_{i=1}^{rN}\lambda_i\,|\,\mathbf{f}^H\mathbf{v}_i\,|^2$ contains contribution from noise.

Since each term is positive, we want each to be minimum for the MSE to be minimum. Observe that for white noise, the third term (noise contribution) becomes $\sigma_w^2\|\mathbf{f}\|^2$. It is insensitive to delay d as long as the norm of the equalizer remains constant, but for colored noise, the contribution is through the relationship between the noise eigenvectors and the equalizer tap vector and the associated eigenvalues. Therefore, even if the norm of the equalizer may remain constant, depending on delay d and the corresponding $\mathbf{f}$, the contribution can change significantly.

Assuming the matrix $\mathbf{H}\mathbf{H}^H + \mathbf{R}_w$ to be positive definite, the Cholesky decomposition provides $\mathbf{H}\mathbf{H}^H + \mathbf{R}_w = \mathbf{L}\mathbf{L}^H$ so that the contribution to the MSE due to the ith column ($i \neq d$) of $\mathbf{H}$ is

$$\left|\mathbf{f}^H\mathbf{h}_i\right|^2 = \left|\left(\mathbf{\Gamma}^H\mathbf{h}_d\right)^H\left(\mathbf{\Gamma}^H\mathbf{h}_i\right)\right|^2 \tag{28}$$

where $\mathbf{\Gamma} = \mathbf{L}^{-1}$. Similarly, the contribution due to the mth noise eigenvector is

$$\lambda_m\left|\mathbf{f}^H\mathbf{h}_{K+m}\right|^2 = \lambda_m\left|\left(\mathbf{\Gamma}^H\mathbf{h}_d\right)^H\left(\mathbf{\Gamma}^H\mathbf{h}_i\right)\right|^2. \tag{29}$$

Equation (27) shows that the contribution to the MSE comes from all the columns of $\tilde{\mathbf{H}}$. Equations (28) and (29) show that the contribution to the MSE from the ith column of $\tilde{\mathbf{H}}$ matrix comes through an inner product with column $\mathbf{h}_d$ (associated with delay d) in a transformed space. For colored noise, the noise eigenvalues are not equal along the eigenvectors. Therefore, for some values of d, the inner product of the dominant eigenvectors with $\mathbf{h}_d$ in the transformed space may contribute large values to the MSE. For some other values of d, the MSE may be very small.

B. Decision Feedback Equalizers (DFEs)

The DFEs behavior in colored noise may also be presented in a similar way as that of a linear equalizer under correct past decision assumption. In this case, the DFE tap vector $\mathbf{f}$ consists of $\mathbf{f}_F$ and $\mathbf{f}_B$, as $\mathbf{f} = [\mathbf{f}_F^T\,\mathbf{f}_B^T]^T$, where $\mathbf{f}_F$ is the fractionally spaced feedforward filter (FFF) tap vector, and $\mathbf{f}_B$ is the symbol spaced feedback filter (FBF) tap vector. The vector $\mathbf{f}_F$ is rN_f taps long, and the vector $\mathbf{f}_B$ is N_b taps long. The minimum mean squared error (MMSE) solution for the tap vector $\mathbf{f}$ is given by [22]

$$\mathbf{f} = \left(\mathbf{H}_{dfe}\mathbf{H}_{dfe}^H + \mathbf{R}_{dfe,\,w}\right)^{-1}\mathbf{h}_d \tag{30}$$

where $\mathbf{H}_{dfe} = [\mathbf{H}_F^T, \mathbf{H}_B^T]^T$. The jth row of matrix $\mathbf{H}_F$ is of the form $[\mathbf{0}_{1,\xi}, \mathbf{h}^{(m)T}, \mathbf{0}_{1,\eta}]$, $\xi = \lfloor j/r \rfloor$, $m = r - 1 - j \bmod r$, $\eta = N_f - 1 - \xi$, $0 \leq j < rN_f$. The mth subchannel response vector $\mathbf{h}^{(m)}$ is as in the case of the linear equalizer. The matrix $\mathbf{H}_B$ equals $[\mathbf{0}_{N_b,\,d+1}, \mathbf{I}_{N_b}]$, and $\mathbf{h}_d$ is the dth column of $\mathbf{H}_{dfe}$. The DFE noise correlation matrix $\mathbf{R}_{dfe,\,w}$ is of the form

$$\mathbf{R}_{dfe,\,w} = \begin{pmatrix}\mathbf{R}_w & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}$$

where $\mathbf{R}_w$ is the correlation matrix of the noise samples at the FFF. For correct past decisions, the MSE can be expressed as

$$\text{MSE} = \left|1 - \mathbf{f}_F^H\mathbf{h}_d\right|^2 + \sum_{i\in Q}^{K}\left|\mathbf{f}_F^H\mathbf{h}_i\right|^2 + \sum_{i\in Q'}^{K}\left|\mathbf{f}_F^H\mathbf{h}_i + f_{B,\,i}\right|^2 + \sum_{i=1}^{rN}\lambda_i\left|\mathbf{f}_F^H\mathbf{v}_i\right|^2 \tag{31}$$

where Q is the set of symbols that contribute to the received signal samples fed to the FFF except a_{n-d} and the symbols considered in the FBF. The set Q' contains all symbols that contribute to the received samples fed to the FFF and are also considered at the FBF.

VI. Simulations and Discussions

The effects of colored noise model due to higher sampling rates is studied for both the channel estimation and equalization algorithms. The signal-to-noise ratio (SNR) is defined as E_b/N_0, where $E_b = \sum_l |u(lT_r)|^2/r$, and N_0 is the noise power spectral density.

A. Performance of Blind Identification under Different Sampling and Prefiltering

We implement the well-known subspace method [7] in our investigation of the discretization effect on the performance of blind identification algorithms. Recall that the subspace method, as in many other approaches, assumes that the noise is white after oversampling. Thus, one important question to answer is whether blind channel identification performs better for white (oversampled) noise, requiring prefilters with wider bandwidth or for colored noise from prefilters with bandwidth equal to the signal bandwidth.

The performance of the subspace method for channel estimation combined with a DFE for data detection is shown in Fig. 3 for different sampling rates with colored noise samples. A data length of 500 symbols is used, and the results are obtained by averaging over 50 independent trials. The transmit filter $p(t)$ is an RRC filter with $\alpha = 0.35$ and rolloff over $1/2T$. The multipath channel $c(t)$ is modeled as consisting of four taps of magnitudes $\{1.0, -0.4, 0.5, 0.3\}$ corresponding to delays $\{0, T/3, 2T/3, T\}$. The overall channel impulse response is

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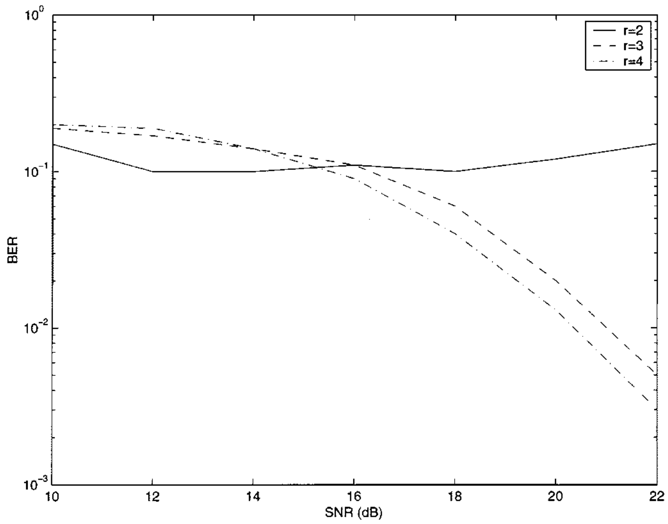

Fig. 3. Performance of the subspace method with a decision feedback equalizer at different sampling rates (r) with colored noise.

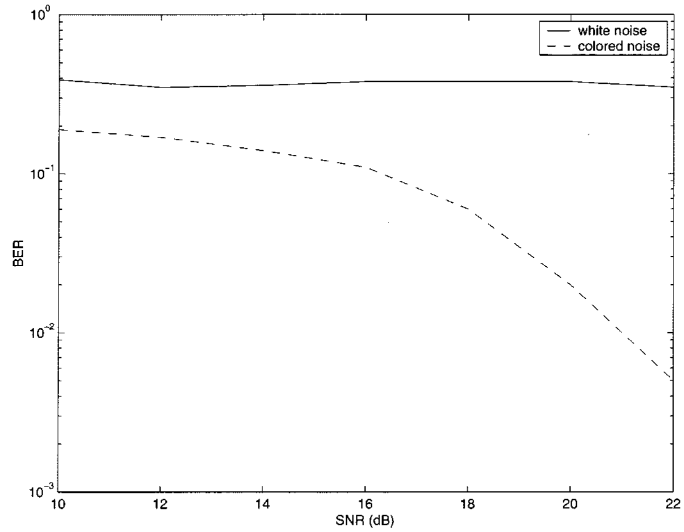

Fig. 4. Performance of the subspace method with a decision feedback equalizer under different noise models for $r = 3$.

$p(t) * c(t) * g(t)$, where $g(t)$ is the receive prefilter. The receive filter $g(t)$ is truncated to five symbol periods. The DFE FFF uses $12r$ taps. The number of FBF taps is $10 + L - d$. Perfect knowledge of the noise autocovariance is assumed, and the FFF/FBF taps are computed using the estimated overall channel impulse response and the noise autocovariance [22]. The figure shows that the presence of the prefilter makes the identification problem difficult for the subspace method since many eigenvalues of the signal subspace are very small. In this example, $r = 3$ shows better performance than $r = 2$. Further increase in the sampling rate does not show marked improvement in performance. It will be seen in Section VI-B that an increase in the sampling rate does not improve the performance of the equalization algorithm. Thus, the improvement in performance in using $r = 3$ compared with $r = 2$ must come from the subspace channel identification algorithm. This performance improvement may be due to the ease of signal and noise subspace separation at $r = 3$ compared with $r = 2$. Note that the subspace method is not an optimal identification algorithm.

Fig. 4 shows the performance of the subspace method with a DFE for white and colored noise at $r = 3$. The prefilter used in the colored noise case has a roll off over $1/2T$ so that the noise samples of the subchannel are white, but the noise samples of different subchannels are correlated. In this case, an improvement in performance in the colored noise case is observed. The white noise case uses a prefilter with a rolloff over $r/2T$ and thus allows more noise at its output.

B. Performance of Equalizers Under Different Discretizations

The role of the prefilter under colored noise model is investigated here. If the noise coloring is due to the prefilter, then the effect of the prefilter must be considered carefully in the overall channel impulse response. A negligible fraction of energy (0.034%) is lost if the overall channel impulse response is truncated to six symbol periods. Fig. 5 shows the MSE versus delay d at an SNR of 15 dB for linear equalizers. The equalizer consists of 60 fractionally spaced taps with a sampling rate of $r = 3$. The figure shows that for white noise, the MSE is

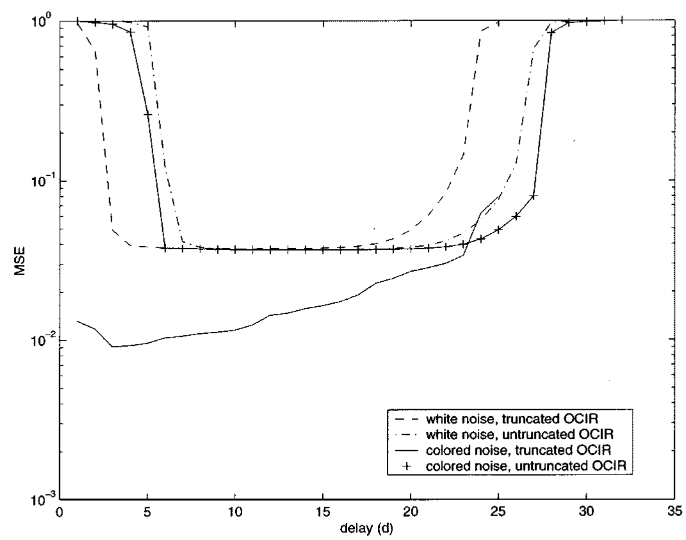

Fig. 5. MSE at different delays for the linear equalizer showing the effect of truncation of the overall channel impulse response.

not very sensitive in the middle region of the allowed delays, but for colored noise, the MSE becomes very sensitive to delay d when the overall channel impulse response (OCIR) is truncated. When the OCIR is not truncated, behavior in white and colored noise are similar. Therefore, in the study of equalizer performance in colored noise due to the prefilter, care must be taken in truncating the overall channel impulse response. This needs further investigation.

Fig. 6 shows the MSE performance of the linear equalizer under different sampling rates. The overall channel impulse response is not truncated. This figure shows that there is no benefit in equalizer performance in going to higher sampling rates.

VII. CONCLUSIONS

This paper shows that the analog prefilter, the sampling rate, and the noise model assumed in the equalization algorithms are closely related and that these should be carefully considered in

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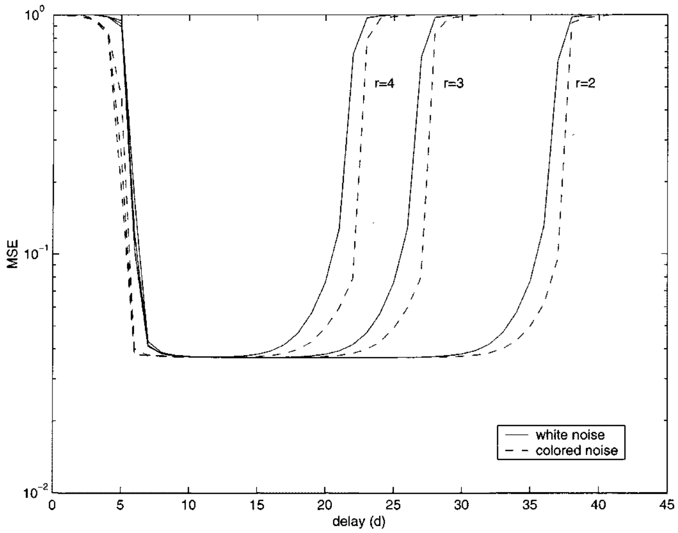

Fig. 6. MSE at different delays for the linear equalizer when different sampling rates are employed.

comparing the performance of different blind algorithms presented in the literature. The fractionally spaced samples at the output of an analog filter, with a spectral support at least equal to that of signal part of the received signal, contain sufficient statistics for several detection criteria. An increase in the sampling rate introduces more noise in an algorithm that assumes a white noise model since the bandwidth of the received filter must be increased to keep the noise white. On the other hand, if the bandwidth of the received filter is kept fixed, an increase in the sampling rate results in colored noise. The effect of colored noise on the performance of linear and decision feedback equalizers under correct past decision assumption is studied, and it is found that in colored noise, the performance is very sensitive to the truncation in the length of the overall channel impulse response. An increase in the sampling rate does not show MSE performance improvement in linear and decision feedback equalizers.

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