Phase-Precoding without CSI for Packet Retransmissions over Frequency-Selective Channels

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***Abstract*—In this paper, we present a simple and effective precoding technique to mitigate inter-symbol interference from multiple transmissions of the same packet in automatic repeat request (ARQ) protocols over slowly time-varying frequency-selective channels. We minimize the power of inter-symbol interference by introducing phase-precoding transmission diversity among subsequent ARQ transmissions. For each packet retransmission, only the phases of the modulated symbols are changed according to a specific pattern. In order to find the best precoding phases assuming that no channel state information is available at the transmitter, we first derive a performance criterion on the precoding phases for a maximum likelihood receiver. Then, we propose a low complexity periodic precoding solution. Simulation results show that the proposed precoding scheme provides a substantial gain in terms of frame error rate performance of the joint detector without significant increase in receiver complexity, leading to enhanced throughput efficiency and better dropping rate in comparison with the non-precoded system.**

***Index Terms*—Automatic repeat request protocols, inter-symbol interference, phase-precoding, turbo-equalization.**

I. Introduction

In high speed data transmission over frequency-selective channels, the received signal suffers from inter-symbol interference (ISI) resulting from a limited bandwidth of the channel or multipath propagation. In order to combat the negative effects of the ISI on the performance of the communication system, advanced signal processing techniques have been introduced. When channel state information (CSI) is available at the transmitter, precoding (pre-equalization) techniques [1], [2] can be used in order to transform the ISI channel into an ISI-free channel. While in the case where no CSI is available, equalization techniques are usually used at the receiver to mitigate ISI from the received signal. In particular, turbo-equalization [3] is an efficient technique that combines signal detection and error correction in an iterative way, leading to a significant performance gain in comparison with systems using separate signal detection and decoding. Even though, errors may occur in the received packets. To ensure data reliability through transmissions, forward error correction coding or automatic repeat request (ARQ) protocols [4] are usually used to combat errors introduced by the communication channel. In Hybrid ARQ (HARQ) protocols, a combination of both methods is used to enhance data throughput. Basically, HARQ protocols have two operational modes: namely Chase combining which is based on the retransmission of the same coded packet, and incremental redundancy which is based on the retransmission of additional redundancy bits. In the context of Chase combining mode, channel equalization performance can be improved by performing joint equalization of all received copies of the same packet [5] compared with separate equalization followed by maximum-ratio-combining. When no channel diversity is available, as in long-term quasi-static channels for example, only an accumulated signal to noise ratio (SNR) gain can be expected after joint equalization. However, system performance can be improved by introducing some transmission diversity among subsequent HARQ transmissions. For example, a mapping diversity scheme was proposed in [6] to increase the Euclidean distance separation between transmitted packets. The drawback of this method is to be limited to high order modulations. Moreover, the optimized mapping depends on many parameters including the actual SNR and the variance of the log-likelihood ratios of the previously decoded packets. These parameters must be fed back to the transmitter resulting in an increased load on the feedback channel and additional memory requirements to store the optimized mappings for quantized values of these parameters. Another transmission diversity scheme is proposed in [7] using a different linear filter-based precoder for each transmission assuming that CSI is known by the transmitter.

In this paper, we present a novel diversity scheme based on phase-precoding to combat the ISI in Chase combining mode of HARQ protocols by changing the phases of the transmitted symbols at each HARQ transmission. An important key feature of this technique is that no CSI knowledge is assumed at the transmitter. We derive a performance criterion on the selection of precoding phases for an optimal maximum likelihood (ML) receiver. To exploit the introduced phase-precoding diversity, we present a low complexity joint soft-input soft-output (SISO) equalizer based on linear filtering under the minimum mean squared error (MMSE) criterion. The SISO MMSE equalizer can be used in an iterative turbo-equalization scheme in order to approaches the performance of the optimal maximum likelihood receiver. However, we show that the performance gain due to the proposed phase-precoding diversity is even better for a linear receiver with separate equalization and decoding.

The main advantages of the proposed precoding technique are summarized by the following points:

- No CSI is required at the transmitter;
- It can be applied for any modulation order;

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- It can be applied for coded or non-coded systems;
- It preserves power characteristics of the modulated signal;
- It provides a substantial performance gain without significant additional complexity.

The remaining of this paper is organized as follows. In Section II, we introduce the system model and the proposed phase-precoding technique. In Section III, we carry out an error probability analysis in order to define a suitable performance criterion for the selection of the precoding phases, and we present the proposed solution. In Section IV, we present the receiver structure using low complexity MMSE equalization that exploits the introduced phase-precoding diversity. In Section V, we give some simulation results showing the efficiency of the proposed precoding technique. Finally, conclusions are given in Section VI.

II. Phase-Precoded HARQ System

We consider the model of a communication system with retransmission shown in Figure 1. A packet of KQ information bits $\mathbf{d} = (d_1 \cdots d_{KQ})$ are encoded by a rate-K/N error correction code to obtain a codeword $\mathbf{c} = (c_1 \cdots c_{NQ})$ of NQ coded bits. After a pseudo-random interleaver Π, the encoded bits are mapped into a sequence of symbols $\mathbf{x} = (x_1 \cdots x_N)$ using a complex modulation alphabet $\S$ of size $|\S| = 2^Q$ symbols with average power E_s assuming that all symbols are transmitted with equal probability. In each HARQ transmission, the same modulated symbol x_n is multiplied by a complex-valued precoding coefficient of unit amplitude $a_n^{(f)} = e^{j\phi_n^{(f)}}$ to obtain the precoded symbol $y_n^{(f)} = a_n^{(f)} x_n$, where f is the index of the HARQ transmission. The precoded symbols $y_n^{(f)}$ are then transmitted through a frequency-selective channel modeled by its equivalent complex-valued discrete-time finite impulse response of length L, denoted by $\mathbf{h}^{(f)} = (h_0^{(f)} \cdots h_{L-1}^{(f)})$ and assumed constant during each transmission but it may slightly vary from one HARQ transmission to the next. The received sequence samples $r_n^{(f)}$ corresponding to the f^{th} HARQ transmission are modeled as

$$r_n^{(f)} = \sum_{i=0}^{L-1} h_i^{(f)} y_{n-i}^{(f)} + w_n^{(f)},\ n = 1, \cdots, N,$$

where $w_n^{(f)}$ is a complex Gaussian noise with variance $\sigma_w^2/2$ per real dimension. At the receiver, we consider a joint detection and decoding scheme assuming perfect CSI. At the current HARQ round F, the receiver estimates the transmitted packet from all received precoded versions of the modulated sequence $\mathbf{x}$. If the packet is still in error after a maximum number $F_{\max}$ of allowable transmissions, an error is declared and the packet is dropped out from the transmission buffer.

The first question we address is how to select the precoding coefficients $a_n^{(f)}$ in order to reduce the effect of the ISI on the frame error rate (FER) performance assuming that the channel does not change between subsequent HARQ transmissions. This is the subject of Section III.

III. Error Probability Analysis

In order to find out the best precoding coefficients, we carry out an error probability analysis for the joint ML receiver. From this analysis, we derive a performance criterion suitable for the choice of the precoding coefficients.

Let $\mathbf{x}$ and $\hat{\mathbf{x}}$ be the transmitted and the estimated sequence, respectively. Let $\mathbf{y}^{(f)}$ and $\hat{\mathbf{y}}^{(f)}$ be the corresponding precoded sequences at the f^{th} transmission. We define the following useful error sequences $\mathbf{e} \triangleq \hat{\mathbf{x}} - \mathbf{x}$ and $\tilde{\mathbf{e}}^{(f)} \triangleq \hat{\mathbf{y}}^{(f)} - \mathbf{y}^{(f)}$. After F HARQ transmissions, the pairwise error probability for the joint ML receiver between $\mathbf{x}$ and $\hat{\mathbf{x}}$, denoted by $P_2(\hat{\mathbf{x}}, \mathbf{x})$, is given in [8] by

$$P_2(\hat{\mathbf{x}}, \mathbf{x}) = Q\left(d_E / 2\sigma_w\right),$$

where $Q(\cdot)$ is the Gaussian error probability function, and d_E is the Euclidean distance separation between $\mathbf{x}$ and $\hat{\mathbf{x}}$ at the output of noiseless ISI channel. The squared Euclidean distance is evaluated as follows

$$d_E^2 = \sum_{f=1}^{F} \sum_{n=1}^{N+L-1} \left| \sum_{i=0}^{L-1} h_i^{(f)} \tilde{e}_{n-i}^{(f)} \right|^2, \quad (1)$$

with $\tilde{e}_n^{(f)} = \hat{y}_n^{(f)} - y_n^{(f)} = a_n^{(f)} e_n$. By developing the squared sum in (1) and performing some algebraic computations, we can rewrite d_E^2 as the sum of two terms as follows

$$d_E^2 = \Gamma_F + \Delta_F, \quad (2)$$

with

$$\Gamma_F \triangleq \sum_{f=1}^{F} R_0^*(\mathbf{h}^{(f)}) R_0(\tilde{\mathbf{e}}^{(f)}), \quad (3)$$

$$\Delta_F \triangleq 2\Re\left[\sum_{f=1}^{F} \sum_{\ell=1}^{L-1} R_\ell^*(\mathbf{h}^{(f)}) R_\ell(\tilde{\mathbf{e}}^{(f)})\right], \quad (4)$$

where $\Re[\cdot]$ denotes the real part of the complex argument, the superscript $(\cdot)^*$ denotes the complex conjugate, and $R_\ell(\cdot)$ is the aperiodic auto-correlation function (ACF) at lag ℓ, defined for an arbitrary complex sequence $\mathbf{x}$ of length N by

$$R_\ell(\mathbf{x}) \triangleq \sum_{n=1}^{N} x_n x_{n-\ell}^*,$$

with $x_n = 0$ for $n \notin [1, N]$. This assumes that the symbols outside the sequence period are known by the receiver and the corresponding error elements are zeros. This can be justified by the use of a training sequence between transmitted packets for channel estimation and synchronization purposes. The obtained expression for the squared Euclidean distance is equivalent to that given by Forney in [8] using polynomial notations. The first term Γ_F takes positive real values and gives the effect of the channel gain on the squared Euclidean distance. Whereas, the second term Δ_F takes signed real values reflecting the fluctuation of the Euclidean distance due to the presence of the ISI. Obviously, for an ISI-free channel, we have $\Delta_F = 0$.

Providing that $|a_n^{(f)}| = 1$, we have $R_0(\tilde{\mathbf{e}}^{(f)}) = R_0(\mathbf{e})$ which means that phase-precoding does not change the squared amplitude of the error sequence. Hence the variable Γ_F is invariant by phase-precoding. Consequently, phase-precoding does not change system performance over ISI-free channels. By contrast, the interference term Δ_F depends on the precoding coefficients through the ACF of the precoded error sequence

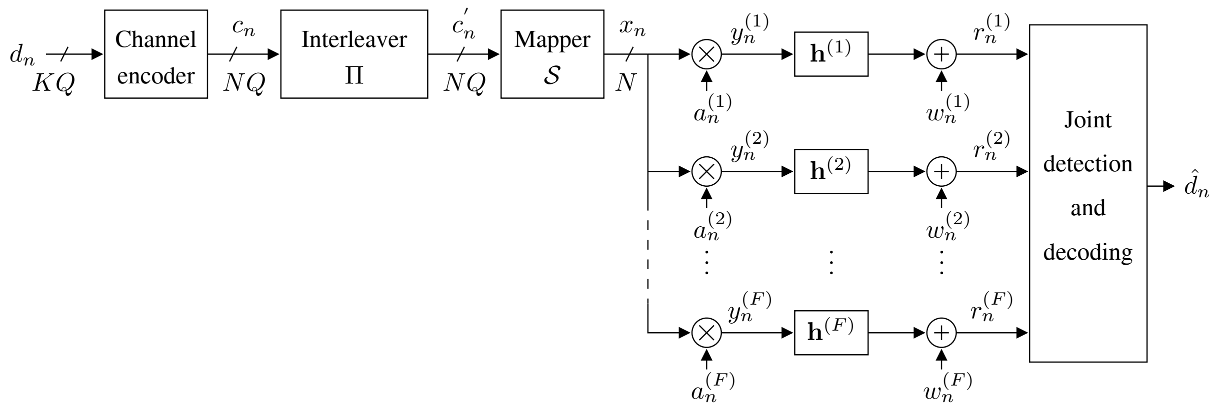

Fig. 1. Phase-precoded HARQ system model.

$R_\ell(\tilde{\mathbf{e}}^{(f)})$. In order to minimize the effects of the ISI on the FER performance, we intend to minimize the interference variable Δ_F with respect to the precoding coefficients in the mean squared error sense. In other words, we minimize the variance of Δ_F for all error sequences leading to a fixed Euclidean distance over an ISI-free channel. To simplify our analysis, we consider the case of long-term quasi-static channels and we generalize our results to slowly time-varying channels through numerical simulations.

A. Performance criterion

In long-term quasi-static channel model, the channel does not change between subsequent HARQ transmissions of the same packet ($\mathbf{h}^{(1)} = \cdots = \mathbf{h}^{(F)} \triangleq \mathbf{h}$), but may change from packet to packet [9]. In this case, equation (4) reduces to

$$\Delta_F = 2\Re\left[\sum_{\ell=1}^{L-1} R_\ell^*(\mathbf{h})\Sigma_{F,\ell}\right], \quad (5)$$

with

$$\Sigma_{F,\ell} \triangleq \sum_{f=1}^{F} R_\ell(\tilde{\mathbf{e}}^{(f)}) = \sum_{n=1}^{N-\ell} C_F(n,\ell)e_n^* e_{n+\ell}, \quad (6)$$

where

$$C_F(n,\ell) \triangleq \mathbf{a}_{F,n}^H \mathbf{a}_{F,n+\ell} = \sum_{f=1}^{F} (a_n^{(f)})^* a_{n+\ell}^{(f)}, \quad (7)$$

which is the cross-correlation between the precoding vectors $\mathbf{a}_n^{(F)}$ and $\mathbf{a}_{n+\ell}^{(F)}$ where $\mathbf{a}_n^{(F)} \triangleq [a_n^{(1)} \cdots a_n^{(F)}]^T$ denotes the vector obtained by regrouping the precoding coefficients of the same symbol x_n during the first F transmissions.

The main idea behind the proposed phase-precoding is to exploit the time diversity in order to separate different interfering paths by orthogonalization of adjacent symbols (within the channel memory L). The interference between L adjacent symbols can be assimilated to the interference between L users in a multiple access system. We can separate different symbols by allocating to each symbol a different orthogonal spreading code in the retransmission dimension. For complete interference mitigation, the code length (here the number of transmissions F) must be at least equal to L. In this particular case, a simple matched filter receiver can efficiently separate different paths without the need for equalization. However, allocating shorter spreading codes would reduce the interference level seen by each symbol. From the Euclidean distance point of view, two valid transmitted sequences having a low Euclidean distance in one transmission are remapped in the next transmission into two sequences with a high Euclidean distance, and vice-versa, in such a way that the overall Euclidean distance after combining is close to its value for an equivalent ISI-free channel. Mathematically, this is expressed by reduced variations of the interference term Δ_F in the Euclidean distance.

To continue our analysis, let $\mathcal{E}_\gamma$ be the ensemble of all error sequences $\mathbf{e}$ between pairs of non-precoded packets separated by a given squared Euclidean distance $\gamma = ||\mathbf{e}||^2$. We assume that the components of the error sequence e_n are modeled as complex-valued independent and identically distributed (i.i.d.) random variables with zero mean. This assumption is obviously verified for non-coded systems. For coded systems, the i.i.d. property is approximately verified thanks to the uniform interleaver. Under this assumption, the interference term Δ_F is considered as a random variable over $\mathcal{E}_\gamma$ with zero mean. In order to determine an objective criterion on the choice of the precoding coefficients, we derive an upper bound on the variance of Δ_F. Then, we minimize the obtained upper bound with respect to the precoding coefficients. The squared value of Δ_F is upper bounded, using the inequality $\Re[x]^2 \leq |x|^2$, as

$$\Delta_F^2 \leq 4\left|\sum_{\ell=1}^{L-1} R_\ell^*(\mathbf{h})\Sigma_{F,\ell}\right|^2, \quad (8)$$

with equality for a real modulation alphabet and a real channel response. Under the i.i.d. assumption for error components, it can be easily shown from (6) that the random variables $\{\Sigma_{F,\ell} : \ell = 1, \cdots, L-1\}$ are pairwise uncorrelated. By developing the squared sum on the right-hand side of (8) and taking the expectation of both sides over $\mathcal{E}_\gamma$ we obtain

$$\mathbb{E}(\Delta_F^2) \leq 4\sum_{\ell=1}^{L-1} |R_\ell(\mathbf{h})|^2 \mathbb{E}(|\Sigma_{F,\ell}|^2), \quad (9)$$

where the expectation $\mathbb{E}[|\Sigma_{F,\ell}|^2]$ can be evaluated from (6) as

$$\mathbb{E}[|\Sigma_{F,\ell}|^2] = \sum_{n=1}^{N-\ell} |C_F(n,\ell)|^2 \mathbb{E}[|e_n^* e_{n+\ell}|^2]. \quad (10)$$

Note that the expectation $\mathbb{E}(|e_n^* e_{n+\ell}|^2)$ is the variance of the product of two i.i.d. random variables with zero mean. Let σ_e^2 denotes the common variance of e_n. Consequently, we have $\mathbb{E}[|e_n^* e_{n+\ell}|^2] = \sigma_e^4$ which is independent of n and ℓ, and therefore can be moved out of the sum in (10) as follows

$$\mathbb{E}[|\Sigma_{F,\ell}|^2] = \sigma_e^4 \lambda_{F,\ell}(\mathbf{A}), \quad (11)$$

where $\mathbf{A}$ denotes the $F_{\max} \times N$ precoding matrix whose n^{th} column is the precoding vector $\mathbf{a}_{F_{\max},n}$, and

$$\lambda_{F,\ell}(\mathbf{A}) \triangleq \sum_{n=1}^{N-\ell} |C_F(n,\ell)|^2, \quad (12)$$

is the total squared cross-correlations between all precoding vectors separated by ℓ positions. By substituting (11) in (9), we obtain

$$\mathbb{E}[\Delta_F^2] \leq 4\sigma_e^4 \sum_{\ell=1}^{L-1} |R_\ell(\mathbf{h})|^2 \lambda_{F,\ell}(\mathbf{A}), \quad (13)$$

which is a weighted sum of the auto-correlation function of the channel indicating that the precoding vectors must be locally uncorrelated within the channel memory L, especially for lags with high channel auto-correlation. Since no CSI is assumed in this paper, we separate the effect of the precoding matrix by applying Cauchy-Schwartz inequality on the right-hand side of (13) to obtain an upper bound on the variance of Δ_F as

$$\mathbb{E}[\Delta_F^2] \leq 4\sigma_e^4 \left(\sum_{\ell=1}^{L-1} |R_\ell(\mathbf{h})|^4\right)^{1/2} \left(\sum_{\ell=1}^{L-1} \lambda_{F,\ell}^2(\mathbf{A})\right)^{1/2}. \quad (14)$$

That is only the last term in the upper bound (14) which depends on the precoding coefficients. Therefore, the minimization of the upper bound with respect to precoding coefficients is equivalent to the minimization of the following cost function

$$J_F(\mathbf{A}) = \left(\sum_{\ell=1}^{L-1} \lambda_{F,\ell}^2(\mathbf{A})\right)^{1/2}. \quad (15)$$

Since the cost function is based on an upper bound on the variance of Δ_F, an optimal solution that minimizes the cost function does not necessarily minimize the variance for a given channel response. However, a variance reduction can be expected regardless of the channel realization.

Minimizing the cost function for a given value of F is a difficult multidimensional optimization problem due to the inter-dependency between the total cross-correlation variables $\lambda_{F,\ell}$. However, some general properties of the optimal solution are found by inspecting the minimum achievable value for the cost function. In fact, applying Cauchy-Schwartz inequality for sums of squares of real numbers $\left(\sum_{k=1}^N b_k\right)^2 \leq N \sum_{k=1}^N b_k^2$ on the expression of the cost function in (15) gives

$$J_F(\mathbf{A}) \geq \frac{1}{\sqrt{L-1}} \sum_{\ell=1}^{L-1} \lambda_{F,\ell}(\mathbf{A}) \triangleq \frac{S_F(\mathbf{A})}{\sqrt{L-1}}, \quad (16)$$

with equality *if and only if* $\lambda_{F,1}(\mathbf{A}) = \cdots = \lambda_{F,L-1}(\mathbf{A})$. Consequently, an ideal solution is a precoding matrix $\mathbf{A}$ which jointly verifies the two following properties:

1) Minimal total cross-correlation $S_F(\mathbf{A})$: This property ensures a maximum precoding gain in average for channels with uniform power-delay profile.
2) Uniform distribution for $\lambda_{F,\ell}(\mathbf{A})$ over lags: This property ensures that some precoding gain can be obtained for any particular channel realization.

Another difficulty arises from the fact that an optimal precoding solution which simultaneously minimizes the cost function for all $F \leq F_{\max}$ may not exist. In this case, some minimization strategy has to be considered. For a particular choice of precoding coefficients, we consider the normalized value of the cost function by its value for the non-precoded system, denoted by G_F, as an *indicator factor* of the goodness of this choice. Hence

$$G_F(\mathbf{A}) \triangleq J_F(\mathbf{A})/J_F(\mathbf{1}), \quad (17)$$

where $\mathbf{1}$ is the $F_{\max} \times N$ matrix with all its elements are 1. The indicator factor takes its values in the interval $[0, 1]$. A smaller value of G_F indicates a better precoding solution.

By the following, we present two sub-optimal solutions, namely *random precoding* and *periodic precoding*. The random precoding solution satisfies the uniform cross-correlation distribution property, whereas the periodic precoding solution minimizes total cross-correlation $S_F(\mathbf{A})$.

B. Random precoding solution

For this solution, we select the precoding coefficients randomly from a finite alphabet $\mathcal{A} = \{e^{2j\pi k/K} : k = 0, \cdots, K-1\}$ consisting of K (for $K \geq 2$) complex numbers uniformly distributed over the unit circle. For large N, the value of $\lambda_{F,\ell}$ defined in (12) can be approximated as

$$\lambda_{F,\ell} \approx (N-\ell)\mathbb{E}[|C_F(n,\ell)|^2] = (N-\ell)F,$$

where the expectation is taken over all possible random selections of the precoding vectors. It follows from (15) that

$$J_F(\mathbf{A}) \approx \left(\sum_{\ell=1}^{L-1} (N-\ell)^2 F^2\right)^{1/2}.$$

By substituting this value in (17), we obtain an approximated value for the indicator factor given by

$$G_F(\mathbf{A}) \approx \frac{\left(\sum_{\ell=1}^{L-1} (N-\ell)^2 F^2\right)^{1/2}}{\left(\sum_{\ell=1}^{L-1} (N-\ell)^2 F^4\right)^{1/2}} = \frac{1}{F},$$

indicating that the cost function for random precoding is F times lower than the non-precoded system. Next, we present a more structured precoding solution based on the minimization of S_F leading to better performance and lower implementation complexity.

C. Periodic precoding solution

To simplify the optimization problem and for equalization complexity reasons as it will be seen later in Section IV, we restrict ourselves to periodic precoding patterns of period $P \leq L$. Initially, let $P = L$. We construct the precoding matrix by selecting a set of L precoding vectors $\{\mathbf{v}_1, \cdots, \mathbf{v}_L\}$ of dimension $F_{\max}$. These vectors are periodically assigned to transmitted symbols, i.e. we assign to the symbol x_n the precoding vector $\mathbf{v}_k$ where $k = (n-1 \bmod L)+1$. We denote by $\mathbf{V}_L$ the $F_{\max} \times L$ matrix whose columns are the precoding vectors $\mathbf{v}_k$. For convenience, we assume the packet length N is an integer multiple of L. The precoding matrix $\mathbf{A}_L$ is obtained by N/L repetition of the generating matrix $\mathbf{V}_L$. In this case S_F can be written in a simpler form as

$$S_F(\mathbf{A}_L) = \sum_{\ell=1}^{L-1}\sum_{n=1}^{N-\ell} |\mathbf{a}_{F,n}^H \mathbf{a}_{F,n+\ell}|^2 = \left(\frac{N}{L} - \frac{1}{2}\right) \sum_{i=1}^{L}\sum_{\substack{j=1\\ j\neq i}}^{L} |\mathbf{v}_{F,i}^H \mathbf{v}_{F,j}|^2 = \left(\frac{N}{L} - \frac{1}{2}\right)\left(\mathrm{TSC}_F(\mathbf{V}_L) - LF^2\right) \tag{18}$$

where $\mathrm{TSC}_F(\mathbf{V}_L)$ denotes the *total-squared-correlation* of the set $\mathbf{V}_L$ taking only into the account the first F components of each vector and defined by

$$\mathrm{TSC}_F(\mathbf{V}_L) \triangleq \sum_{i=1}^{L}\sum_{j=1}^{L} |\mathbf{v}_{F,i}^H \mathbf{v}_{F,j}|^2, \tag{19}$$

which is extensively studied in the literature in the context of code division multiple access systems (see [10] and references therein). It is known that the TSC for a complex-valued set is lower bounded by Welch's bound [11] given by

$$\mathrm{TSC}_F(\mathbf{V}_L) \geq \begin{cases} LF^2 & \text{for} \quad F \geq L \\ L^2 F & \text{for} \quad F \leq L \end{cases} \tag{20}$$

This yields to a lower bound for S_F given by

$$S_F(\mathbf{A}_L) \geq \begin{cases} 0 & \text{for} \quad F \geq L \\ (N - \frac{L}{2})F(L-F) & \text{for} \quad F \leq L \end{cases} \tag{21}$$

Combining (16), (17), and (21) leads to a lower bound for G_F under the periodic constraint as follows

$$G_F(\mathbf{A}_L) \geq \begin{cases} 0 & \text{for} \quad F \geq L \\ \frac{L-F}{(L-1)F} & \text{for} \quad F \leq L \end{cases} \tag{22}$$

We see that the indicator factor G_F is an increasing function with L, and for large values of L compared to F, the indicator factor tends to the value $1/F$ obtained with the random precoding solution.

In the case of a long channel, we show that limiting the period P to any integer divider of L will not change the value of the lower bound as long as $P \geq F_{\max}$. Let $L = \alpha P + \beta$ for some positive integers α and β. Let $\mathbf{A}_P$ be the precoding matrix of period P and $\mathbf{V}_P$ be the corresponding generating matrix. We can evaluate S_F as

$$S_F(\mathbf{A}_P) = \underbrace{\sum_{\ell=1}^{\alpha P-1}\sum_{n=1}^{N-\ell} |\mathbf{a}_{F,n}^H \mathbf{a}_{F,n+\ell}|^2}_{T_1} + \underbrace{\sum_{\ell=\alpha P}^{L-1}\sum_{n=1}^{N-\ell} |\mathbf{a}_{F,n}^H \mathbf{a}_{F,n+\ell}|^2}_{T_2}$$

It can be shown in a similar manner to (18) that the first term T_1 can be expressed as

$$T_1 = \left(\frac{N}{\alpha P} - \frac{1}{2}\right)\left(\alpha^2 \mathrm{TSC}_F(\mathbf{V}_P) - \alpha P F^2\right)$$

When P is an integer divider of L (i.e. $\beta = 0$), the second term T_2 is zero and the achievable lower bound for $S_F(\mathbf{A}_P)$ is the same as for $S_F(\mathbf{A}_L)$ if $P \geq F$, because in this case, $\alpha^2 \mathrm{TSC}_F(\mathbf{V}_P)$ and $\mathrm{TSC}_F(\mathbf{V}_L)$ have the same lower bound $\alpha^2 P^2 F = L^2 F$. When $\beta > 0$, we do not have an explicit lower bound on T_2, but its relative impact on the lower bound of $S_F(\mathbf{A}_P)$ is small when α is large. This explains that the system performance become less sensitive to the precoding period for a long channel response. In this case the precoding period can be taken equal to the maximum number of HARQ transmissions $F_{\max}$. In conclusion, for a known channel length, the precoding period can be chosen as the smallest value $P = L/\alpha \geq F_{\max}$ without increasing S_F, and for unknown channel length, we can take $P = F_{\max}$ with some increase of S_F that vanishes with increasing values of L. The price to pay for reducing the precoding period is a non-uniform distribution of $\lambda_{F,\ell}$ (property 2) because $\lambda_{F,\ell}$ takes the same value as for a non precoded system for any value of ℓ which is multiple of P. In the worst case where all channel delays are multiple of P, no precoding gain can be expected. However, the probability of a such channel realization depends on the channel statistics and it is very small in general. In the case of a short channel length such that $L < P$, the precoding gain tends to zero when L tends to 1 which is natural because the channel becomes less frequency-selective. Therefore, P must be chosen as small as possible in order to take account for short channel realizations.

Finding a set $\mathbf{V}_P$ which simultaneously meets Welch's bound in (20) with equality (WBE) for all F depends on the system parameters P, $F_{\max}$, and the precoding alphabet. For this purpose, we start by rewriting the TSC in (19) in a more convenient form. Let $\mathbf{u}_1, \cdots, \mathbf{u}_{F_{\max}}$ denote the rows of the precoding matrix $\mathbf{V}_P$. It follows from the row-column equivalence property of the TSC [12] that

$$\mathrm{TSC}_F(\mathbf{V}_P) = \sum_{i=1}^{F}\sum_{j=1}^{F} |\mathbf{u}_i \mathbf{u}_j^H|^2. \tag{23}$$

From this equivalence relationship, it was proved in [12] that the necessary and sufficient condition for a set to be WBE that the lines or the columns of the set are orthogonal. The advantage of using a WBE set is that the interference power is uniform across all received symbols. We distinguish two cases:

a) Case $P \geq F_{\max}$: This is usually the case because $F_{\max} \leq 4$ in most practical systems. The set $\mathbf{V}_P$ is a WBE set for all $F \leq F_{\max}$ if all vectors $\mathbf{u}_1, \cdots, \mathbf{u}_{F_{\max}}$ are orthogonal. These vectors could be taken for example from discrete Fourier transform (DFT) matrix of order P or more simply from Hadamard Matrix with bipolar alphabet of order P (when it exists).

b) Case $P < F_{\max}$: In this case, it is not possible to have a WBE set for all $F \leq F_{\max}$ because adding any $m < P$ vectors to a WBE set results in a set which has no longer the

WBE property [10]. Therefore, some optimization strategy has to be considered. We consider in priority the minimization of the cost function at the early retransmissions. This enhances the throughput efficiency of the system. In fact, a WBE is still possible, at least for any $F \leq P$, by choosing $\mathbf{u}_1, \cdots, \mathbf{u}_P$ from orthogonal bases in $\mathbb{C}^P$. For $F > P$, we complete this set by periodical repetition of the previous vectors up to $F_{\max}$. Note that for $F \geq P$, the lower bound on G_F is zero. The proposed solution achieves this bound for any value of F which is integer multiple of P because we have a complete orthogonality between the column vectors. For these particular values of F, the interference is completely canceled.

1) Fourier-based solution: For example, in the case of $P > F_{\max}$, and as previously mentioned, the generating matrix $\mathbf{V}_P$ can be obtained by selecting $F_{\max}$ rows from the Fourier transform matrix of order P,

$$\mathbf{V}_P = \begin{bmatrix} 1 & e^{j\frac{2\pi}{P}k_1} & \cdots & e^{j\frac{2\pi}{P}(P-1)k_1} \\ 1 & e^{j\frac{2\pi}{P}k_2} & \cdots & e^{j\frac{2\pi}{P}(P-1)k_2} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & e^{j\frac{2\pi}{P}k_{F_{\max}}} & \cdots & e^{j\frac{2\pi}{P}(P-1)k_{F_{\max}}} \end{bmatrix}, \quad (24)$$

where the $k_i \in [0, P-1]$ are the indices of the selected rows. In this case, the phase-precoding of the f^{th} transmission is equivalent to a simple frequency shift of k_f/P. Of course, an arbitrary selection of rows leads to a WBE set, but the best selection is that which gives the most uniform distribution for $\lambda_{F,\ell}(\mathbf{V}_P)$. A very useful property of this structured solution is that the product between any two vectors depends only on the shift between them

$$\mathbf{v}_{F,i}^H \mathbf{v}_{F,i+\ell} = \sum_{f=1}^{F} e^{j2\pi k_f \ell/P}, \quad (25)$$

where the shift ℓ is taken modulo P. As it will been seen later, this property allows reducing the complexity of the receiver. Consequently, the total correlation can be computed as

$$\lambda_{F,\ell}(\mathbf{V}_P) = (N-\ell)\left|\sum_{f=1}^{F} e^{j2\pi k_f \ell/P}\right|^2, \ \ell = 1, \cdots, P-1 \quad (26)$$

which simplifies the evaluation of the cost function. An exhaustive search is performed to find an optimal selection of the DFT lines' indices $\{k_1, \cdots, k_{F_{\max}}\}$ that minimizes the cost function J_F simultaneously for $F = 2, \cdots, F_{\max}$. For example, we find for $F_{\max} = 4$ that a possible solution (which is not unique) is $\{0, 1, 2, 3\}$ for $P = 4$, $\{0, 2, 3, 1\}$ for $P = 5$, and $\{0, 3, 2, 6\}$ for $P = 8$.

2) Hadamard-based solution: In the previous analysis, we did not impose any constraint on the precoding phases. It is sometimes preferable for practical reasons to choose the precoding phases from a limited alphabet. We show in the following that the precoding alphabet has small impact on the precoding gain. The simplest form of phase-precoding is when the precoding alphabet is constrained to have bipolar values (± 1). For bipolar vectors, Welch's bound is only tight for vectors whose number is multiple of 4 and loose otherwise. In that case, we can use Hadamard matrix for the construction of the precoding matrix in the same manner as Fourier Matrix.

TABLE I
PERFORMANCE PARAMETERS OF THE PRECODING MATRIX $\mathbf{A}$ CONSTRUCTED FROM THE BIPOLAR SET $\mathbf{V}_5$.

F	$\lambda_{F,\ell}(\mathbf{A})/\lambda_{F,\ell}(\mathbf{1})$				$S_F(\mathbf{A})/S_F(\mathbf{1})$	G_F
▼	$\ell=1$	$\ell=2$	$\ell=3$	$\ell=4$	▼	▼
1	1.0	1.0	1.0	1.0	1.0	1.0
2	0.2	0.6	0.6	0.2	0.4	0.45
3	0.29	0.11	0.11	0.29	0.2	0.22
4	0.1	0.1	0.1	0.1	0.1	0.1

In the frequency-domain this is equivalent to the superposition of two symmetrically shifted versions of the signal. In general, a tight bound on the TSC for bipolar vectors is given by Karystinos in [13] and can be used in order to find the corresponding lower bound on the reduction factor G_F. For example, for $P = L = 5$ and $F_{\max} = 4$, a direct application of Karystinos's lower bound to our case ([13] Table II, case $P \equiv 1 \bmod 2$) leads to

$$\text{TSC}_F(\mathbf{V}_P) \geq P^2 F + (F-1)F,$$

and the lower bound on G_F becomes

$$G_F(\mathbf{A}_P) \geq \frac{P-F}{(P-1)F} + \frac{F-1}{P(P-1)F},$$

where the second term gives the relative increase of G_F compared to its value for a WBE set. Numerically, we obtain for a WBE set $G_F \geq$ 0.3750, 0.1667, and 0.0625 for F =2, 3, and 4, respectively. Whereas, for a bipolar set meeting Karystinos's bound, we obtain $G_F \geq$ 0.4, 0.2, and 0.1 for F =2, 3, and 4, respectively. We propose the following precoding generator matrix whose lines are quasi-orthogonal,

$$\mathbf{V}_5 = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & -1 & +1 & +1 \end{bmatrix}. \quad (27)$$

We can verify that the bipolar set $\mathbf{V}_5$ achieves Karystinos's lower bound as shown in Table I where we can see that the uniform distribution of $\lambda_{F,\ell}$ is only verified for $F = 4$. In this case the effective value of G_F meets its lower bound.

To show the effect of phase-precoding on the Euclidean distance spectrum, we have simulated the normalized squared Euclidean distance distribution $d_E^2/\Gamma_F = 1 + \Delta_F/\Gamma_F$ for input error sequences with a fixed Hamming weight w. We assume that the non-zero error elements are uniformly distributed over the packet. We consider multiple HARQ transmissions over the Proakis-C ISI channel [14] of length $L = 5$ whose the impulse response is $\mathbf{h} = (0.227, 0.460, 0.688, 0.460, 0.227)$ using binary phase shift keying (BPSK) modulation and the precoding set $\mathbf{V}_5$ defined in (27) . Figure 2 shows simulation results over 10^4 packets with $N = 600$ and $w = 10$ for F =1, 2, 3, and 4. We remark spectrum thinning phenomena with relative variance reductions of 0.32, 0.23, and 0.10 for F =2, 3, and 4, respectively.

In order to exploits the introduced transmission diversity, all received copies of the same packet must be jointly processed by the receiver. To this end, we present in Section IV the receiver structure for an iterative detection and decoding approach. The separate detection and decoding approach follows immediately as a particular case.

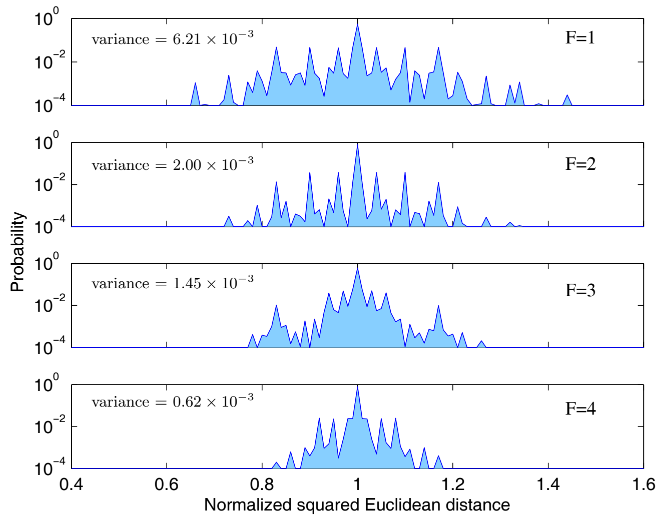

Fig. 2. Normalized squared Euclidean distance distribution at the output of the Proakis-C ISI channel for input error sequences of length $N = 600$ of Hamming weight $w = 10$ using BPSK modulation and bipolar precoding alphabet for F =1,2,3 and 4.

IV. Receiver Structure

We consider the joint turbo-receiver shown in Fig 3 using two soft-input soft-output (SISO) modules for sequence detection and MAP decoding which are connected iteratively as in a classical turbo-equalization scheme. All received signals are first processed by a joint detector which produces the *a posteriori* probabilities $\text{APP}_e(\mathbf{x})$ of the transmitted symbols $\mathbf{x}$. After soft demapping, the extrinsic log-likelihood ratios (LLRs) $L_e(\mathbf{c}')$ of the interleaved binary codeword are obtained by subtracting the interleaved extrinsic LLRs $L_d(\mathbf{c}')$ produced by the channel decoder in the previous iteration. These LLRs are then de-interleaved and processed by the MAP channel decoder to produce the output LLRs which are fed back to the joint detector through the interleaver and the soft mapper for the next turbo-iteration.

We present two different schemes for the joint detector that differ in the way of performing the combining of the various received packets. The first performs input-combining, while the second performs output-combining. For low complexity requirements, we focus on MMSE-based equalization.

A. Joint MMSE equalization (JE)

For each received sequence, the receiver performs the inverse precoding operation to obtain

$$\begin{aligned}\tilde{r}_n^{(f)} &= (a_n^{(f)})^* r_n^{(f)} \\ &= \sum_{i=0}^{L-1} (a_n^{(f)})^* h_i^{(f)} a_{n-i}^{(f)} x_{n-i} + (a_n^{(f)})^* w_n^{(f)} \\ &= \sum_{i=0}^{L-1} \tilde{h}_{n,i}^{(f)} x_{n-i} + \tilde{w}_n^{(f)}.\end{aligned}$$

By considering the phase-precoding as part of the ISI channel, the equivalent ISI channel becomes time-variant whose impulse response during the f^{th} transmission of the symbol x_n is given by

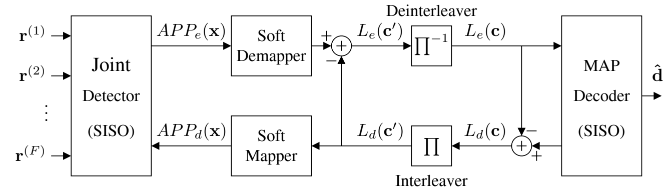

Fig. 3. Turbo-equalizer structure.

$$\tilde{h}_{n,i}^{(f)} \triangleq (a_n^{(f)})^* a_{n-i}^{(f)} h_i^{(f)}. \tag{28}$$

We generalize the finite length MMSE equalizer with *a priori* proposed in [15], [16] to the case of precoded system. The joint SISO MMSE equalizer includes multiple forward linear filters $\mathbf{p}_n^{(f)}$ and an interference canceller filter $\mathbf{q}_n$. The linear estimate $\hat{x}_n$ of the transmitted symbol x_n after F transmissions is given by

$$\hat{x}_n = \sum_{f=1}^{F} (\mathbf{p}_n^{(f)})^H \mathbf{r}_n^{(f)} - \mathbf{q}_n^H \bar{\mathbf{x}}_n, \tag{29}$$

where the superscript $(\cdot)^H$ denotes the hermitian transpose, $\mathbf{r}_n^{(f)} = [r_{n-n_2}^{(f)} \ \cdots \ r_{n+n_1}^{(f)}]^T$ are the required observations samples around the estimated symbol. The forward filters $\mathbf{p}_n^{(f)}$ are implemented using $N_p = n_1 + n_2 + 1$ taps, where the parameters n_1 and n_2 specify the length of the non-causal and the causal part of the estimator filter, respectively. Note that we allow the filter coefficients to vary with n because of the variant-time equivalent channel model defined in (28), and not because we are looking for a time varying solution.

The problem of the joint equalization can be turned back to the case of a single transmission by considering the equivalent single-input multiple-output (SIMO) channel model given in matrix form by

$$\tilde{\mathbf{r}}_n = \tilde{\mathbf{H}}_n \mathbf{x}_n + \tilde{\mathbf{w}}_n,$$

where

$$\begin{aligned}\mathbf{x}_n &= [x_{n-n_2-L+1} \ \cdots \ x_{n+n_1}]^T, \\ \tilde{\mathbf{r}}_n &= [\tilde{r}_{n-n_2}^{(1)} \ \cdots \ \tilde{r}_{n-n_2}^{(F)} \ \cdots \ \tilde{r}_{n+n_1}^{(1)} \ \cdots \ \tilde{r}_{n+n_1}^{(F)}]^T, \\ \tilde{\mathbf{w}}_n &= [\tilde{w}_{n-n_2}^{(1)} \ \cdots \ \tilde{w}_{n-n_2}^{(F)} \ \cdots \ \tilde{w}_{n+n_1}^{(1)} \ \cdots \ \tilde{w}_{n+n_1}^{(F)}]^T,\end{aligned}$$

and $\tilde{\mathbf{H}}_n$ is the $FN_p \times (N_p + L - 1)$ equivalent channel matrix given by

$$\tilde{\mathbf{H}}_n = \begin{bmatrix} \tilde{\mathbf{h}}_{n-n_2,L-1}^{(F)} & \cdots & \tilde{\mathbf{h}}_{n-n_2,0}^{(F)} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \tilde{\mathbf{h}}_{n+n_1,L-1}^{(F)} & \cdots & \tilde{\mathbf{h}}_{n+n_1,0}^{(F)} \end{bmatrix} \tag{30}$$

where $\tilde{\mathbf{h}}_{n,\ell}^{(F)} = [\tilde{h}_{n,\ell}^{(1)} \cdots \ \tilde{h}_{n,\ell}^{(F)}]^T$. Using the equivalent SIMO model, the estimated symbol in (29) can be rewritten as

$$\hat{x}_n = \mathbf{p}_n^H \tilde{\mathbf{r}}_n - \mathbf{q}_n^H \bar{\mathbf{x}}_n. \tag{31}$$

Following the same analysis as in [17], the derivation of the expression of the filters that minimize the mean squared error

$E[|\hat{x}-x_n|^2]$ is straightforward and leads to the following solution

$$\mathbf{p}_n = (\sigma_w^2 \mathbf{I}_{FN_p} + v^2 \tilde{\mathbf{H}}_n \tilde{\mathbf{H}}_n^H)^{-1} \tilde{\mathbf{H}}_n \mathbf{u},$$
$$\mathbf{q}_n = \tilde{\mathbf{H}}_n^H \mathbf{p}_n - \mu_n \mathbf{u},$$

where $\mu_n = \mathbf{p}_n^H \tilde{\mathbf{H}}_n \mathbf{u}$, $v^2 = \frac{1}{N}\sum_{n=1}^N \mathrm{var}(\bar{x}_n)$ is the reliability of the decoder feedback, with $v = 0$ for a perfect feedback, and $v = 1$ for no *a priori*, and $\mathbf{u} = [\mathbf{0}_{1\times(n_2+L-1)}\ 1\ \mathbf{0}_{1\times n_1}]^T$. The output extrinsic a posteriori probabilities (APPs) are then calculated using the Gaussian model for the estimated symbols $\hat{x}_n = \mu_n x_n + \eta_n$, where η_n is a complex Gaussian noise with variance $\nu^2 = \mu_n(1-\mu_n v^2)$.

$$\mathrm{APP}(x_n = x) = K_n \exp\left(-\frac{|\hat{x}_n - \mu_n x|^2}{\nu^2}\right),$$

where K_n is a normalization constant chosen to have a true probability mass function $\sum_{x\in\S} \mathrm{APP}(x_n = x) = 1$ at the output of the estimator. Note that at the first iteration, we have $\bar{\mathbf{x}}_n = 0$ and $v^2 = 1$. The performance of the system for non-iterative equalization and decoding are given by the system performance after the first iteration.

B. Separate equalization with maximum-ratio-combining (SE-MRC)

Another alternative for packet combining with lower complexity is to use a separate equalizer for each transmission followed by maximum-ratio-combiner before the channel decoder. For each transmission, a SISO MMSE equalizer, as described in Section IV-A, with single input is used to detect the precoded sequence $y_n^{(f)}$. Then, the inverse precoding operation is performed after the equalizer by $\hat{x}_n^{(f)} = (a_n^{(f)})^* \hat{y}_n^{(f)}$. The various estimated sequences $\hat{x}_n^{(f)}$ in all transmissions are then combined by a maximum-ratio-combiner operating at the bit level after the soft demapper [18]. At each retransmission, the combiner simply accumulates the extrinsic LLRs $L_e(\mathbf{c}')$ which are de-interleaved and decoded. This type of combining has in general lower performance than joint equalization, but the performance loss is not very important when the residual interferences in the combined signals are uncorrelated. Thanks to the phase-precoding, the non-correlation property is approximately verified. The main advantage of this solution is to be independent of the precoding solution and its period. This resolves the problem of the dependency between the system complexity and the precoding period encountered by the joint equalizer.

It is important to note that the precoding gain results from packet combining and not from the iterative structure of the equalizer. Actually, the phase-precoding decorrelates the ISI among the different received copies in order to add destructively after combining. Consequently, the role of the proposed phase-precoding is to help the equalizer in its task by removing a part of the ISI. This enhances the overall performance for a non-iterative detection approach. Using a powerful detection scheme as a turbo-equalizer could be sufficient alone without the help the phase-precoding in order to remove the interference, but this may require many turbo-iterations. In this case the use of the phase-precoding technique reduces the number of turbo-iterations which are required by the turbo-equalizer to converge.

C. Complexity issues

We discuss now the required additional complexity due to the phase-precoding in comparison with the non-precoded system. In the case of the JE scheme, the complexity of the MMSE equalizer itself per transmission is mainly dominated by the inversion of the matrix $(\sigma_w^2 \mathbf{I} + v^2 \tilde{\mathbf{H}}_n \tilde{\mathbf{H}}_n^H)$ which grows linearly with the number of HARQ transmissions. Since the phase-precoding transforms the ISI channel into a time-variant channel, one matrix inversion is required for each symbol in the frame. Therefore, the complexity of the receiver is highly increased. This is true in general for a non-structured phase-precoding solution like a random phase-precoding. By contrast, for a periodic precoding solution with period P, the required number of matrix inversions is reduced to only P inversions. As we have seen in section III-C that the period value can be chosen as small as $\min(L, F_{\max})$, this significantly reduces the additional complexity. In addition, by using the periodic precoding solution based on the DFT matrix, only one matrix inversion is required because the equivalent channel is actually invariant with n. This results from the particular structure of the precoding coefficients where

$$\tilde{h}_{n,i}^{(f)} = (a_n^{(f)})^* a_{n-i}^{(f)} h_i^{(f)} = e^{-j2\pi \frac{i}{P} k_f} h_i^{(f)}. \qquad (32)$$

In the case of the SE-MRC scheme, the additional complexity is reduced to complex multiplications at the receiver for any phase-precoding solution. Moreover, with a bipolar precoding, the precoding operation and its inverse reduce to simple sign inversion operations.

Finally, in the case of a long channel response, the complexity of the time-domain MMSE equalizer [19] becomes very high due to the large dimension of the channel matrix. It would be interesting to perform the equalization in the frequency-domain with a cyclic-prefix insertion at the transmitter. In this case, the DFT-based precoding turns into a simple cyclic shift in the frequency-domain.

V. Computer Simulations

In order to illustrate the effectiveness of the proposed phase-precoding diversity for HARQ transmissions, we present in the following some simulation results using different system configurations. In the presented simulations, we use a rate-1/2 recursive systematic convolutional code whose generator polynomial is $(1, 21/37)$ in octal notations. A maximum of $F_{\max} = 4$ HARQ transmissions is assumed. We evaluate the system performance by Monte-Carlo simulations versus the average SNR defined as $E_s/N_0 = E_s/\sigma_w^2$. Simulations were performed over a maximum of 10^4 packets. In order to compare the performance of the precoded system to the performance of the non-precoded system under the two proposed detection schemes without turbo-iteration, we first consider a communication system using BPSK modulation over the Proakis-C channel, which is a highly frequency-selective static channel. Since both of the channel response and the modulation alphabet are real, we use the Hadamard-based precoding solution of period $P = L = 5$. This prohibits the exploitation of the imaginary dimension. The MMSE equalizer

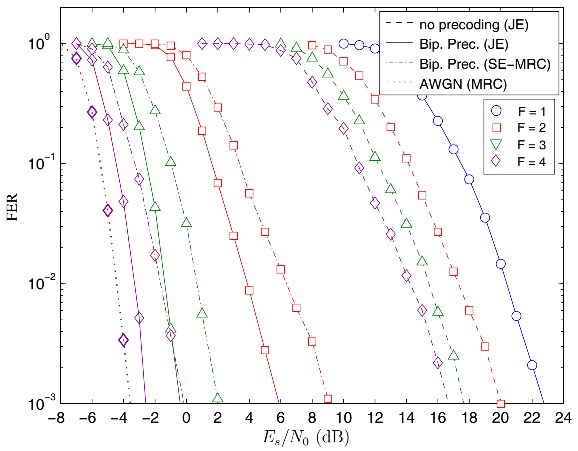

Fig. 4. FER performance of the precoded HARQ system over the Proakis-C frequency-selective channel using BPSK modulation and joint MMSE equalization without turbo-iteration.

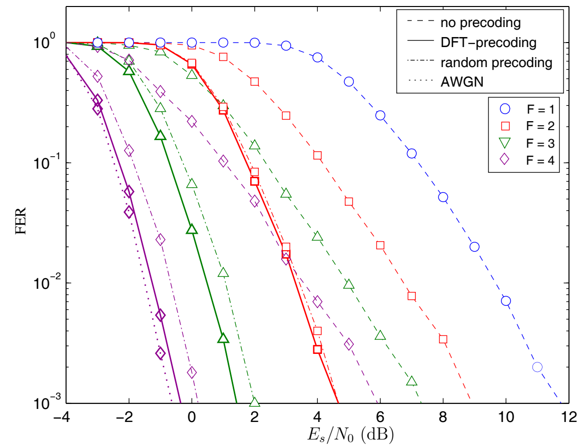

Fig. 5. FER performance of the precoded HARQ system over long-term quasi-static frequency-selective channel ($\rho = 1$) using QPSK modulation. The receiver was implemented using the joint equalization scheme without turbo-iteration.

is implemented using linear filters of length $N_p = 15$ ($n_1 = 9$, $n_2 = 5$). Figure 4 shows the corresponding FER performances.

For the first transmission, the FER performance of the precoded system are the same as for the non-precoded system because phase-precoding does not offer any advantage for a single transmission. For the following retransmissions, a noticeable gain can be observed for both combining schemes. Moreover, the performance of the precoded system for $F = 4$ are close to the system performance over AWGN channel. This indicates that ISI is efficiently removed from the last retransmission resulting in a better dropping rate in the HARQ protocol. We can see clearly that the performance loss of the SE-MRC combining scheme in comparison with the JE scheme is small when compared to the precoding gain.

Now, we consider the transmission system using QPSK modulation with Gray mapping over a random frequency-selective channel with uniform power-delay profile. The channel changes independently from one packet to the next, but stays correlated between successive HARQ retransmissions of the same packet. The correlation coefficient ρ between two subsequent HARQ transmissions is given according to Jakes' model [20] by $\rho = J_0(2\pi f_d \tau)$, where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, f_d denotes the maximum Doppler frequency, and τ is the time delay between two subsequent HARQ transmissions. In our simulations, the channel is normalized to unit energy as in [5], [6] in order to evaluate the effect of ISI on system performance independently of the fading distribution. However, a realistic simulations without normalization are given at the end of this section. Figure 5 compares between a DFT-precoding solution of period $P = L$ and a random precoding solution for $f_d = 0$ ($\rho = 1$). As predicted by our analysis, we can see that the advantage of the DFT precoding with the increasing number of retransmissions. We have found that the performances of the bipolar precoding (not shown on the figure) are only 0.2 dB behind the performance of the DFT precoding reflecting the small effect of the precoding alphabet. By comparing the slope of the FER curve between the precoded and the non-precoded scheme, we observe a diversity gain due to the precoding technique. Figure 6 shows the corresponding data throughput of the HARQ system for $f_d\tau = 0$ and $f_d\tau = 0.1$ ($\rho = 0.9$). We have found by simulation that the throughput of the precoded system stays practically unchanged for all values of ρ. We note that the throughput performance for low to medium SNR values are the same as for AWGN channel. For high SNR values the throughput is essentially dominated by the FER of the first transmission, hence there is no significant improvement in comparison with non-precoded system.

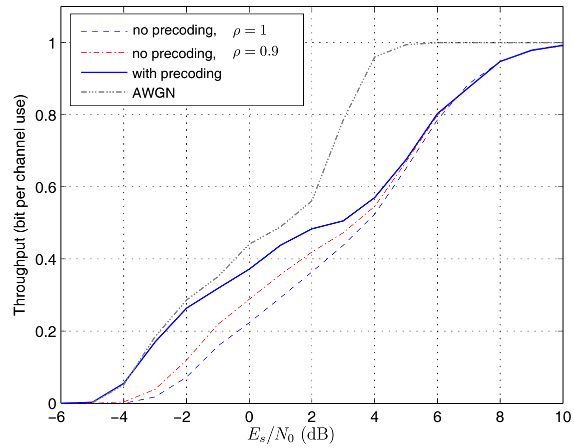

Fig. 6. Throughput of precoded HARQ system over correlated frequency-selective channels for various values of channel correlation coefficient ρ using the DFT-based precoding solution of period $P = 4$ and QPSK modulation.

In addition to the performance gain, the proposed phase-precoding technique improve the convergence behavior when turbo-equalization scheme is used. Fig 7 shows the FER at each turbo-iteration for the case of two HARQ transmissions ($F = 2$). We note that the convergence of the turbo-equalizer for the precoded system is faster than for the non-precoded system thanks to the reduced interference power. The non-

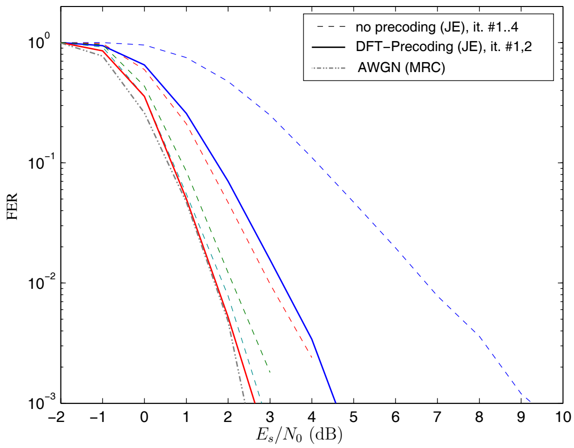

Fig. 7. Convergence behavior of the MMSE turbo-equalizer for the precoded HARQ system over a long-term quasi-static frequency-selective channel using the DFT-based precoding solution of period $P = 4$ and QPSK modulation.

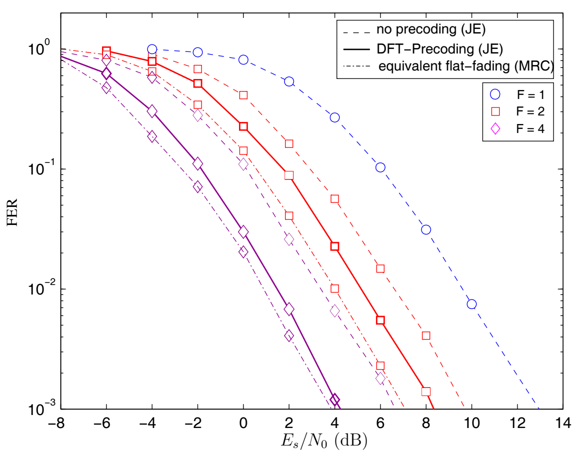

Fig. 8. Phase-precoding performance over the SCME channel model using the DFT-based precoding solution with period $P = 4$.

precoded system needs more than four turbo-iterations to converge, while only two turbo-iterations are required when using phase-precoding.

Finally, we consider a more realistic channel model based on the the 3GPP Spatial Channel Model Extended (SCME) of the European WINNER framework as specified in [21], [22]. This channel model is characterized by six non-zero taps with random delays per link. For each transmitted packet, a random channel realization is generated and used for all HARQ transmissions of the packet. Note that we do not normalize the channel in this case for a more realistic gain evaluation. Other simulation parameters are taken from the 3GPP LTE (Long-Term-Evolution) standard [23]. The transmission speed is $Fs = 7.68$ MSPS, $N = 512$ and the maximum channel delay spread is $L_{\max} = 128$ symbols. Since the channel length is unknown we choose the precoding period $P = F_{\max}$. Due to the long channel memory, the equalizer was implemented in the frequency-domain thanks to cyclic prefix insertion at the transmitter. Fig 8, shows the obtained results where about 2 dB of gain is obtained by phase-precoding at the fourth transmission.

VI. Conclusions

We presented in this paper an efficient phase-precoding technique to mitigate inter-symbol interference from multiple HARQ transmissions over slowly time-varying frequency-selective channels. The introduced phase-precoding technique can be viewed as a transmission diversity technique to combat the channel selectivity in the frequency-domain. A general framework was introduced assuming no CSI is available at the transmitter to find a performance criterion on the precoding coefficients. We proposed an efficient periodic precoding solution leading to a significant gain in FER performance without any significant increase in receiver complexity. The effect of the precoding period and the precoding alphabet and on the precoding gain were investigated. When a turbo-equalization scheme is used at the receiver, the proposed technique allows a faster convergence resulting in a reduced overall complexity. Finally, for a specific channel model with known auto-correlation statistics, phase-precoding technique can further be optimized to enhance system performance.

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